Topological magnetic solitons as a connecting link between curvature and topology

Volodymyr P. Kravchuk

v.kravchuk@ifw-dresden.de
vkravchuk@bitp.kiev.ua

Leibniz-Institut für Festkörper- und Werkstoffforschung (Dresden)
http://www.ifw-dresden.de/

Bogolyubov Institute
for Theoretical Physics (Kyiv, Ukraine)
http://bitp.kiev.ua/

Contents of the talk
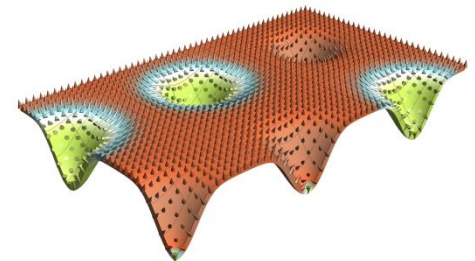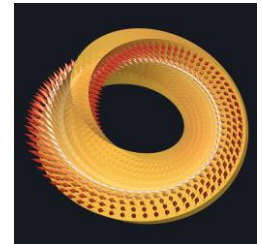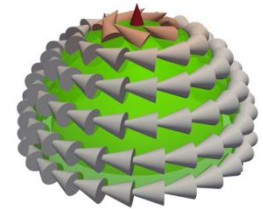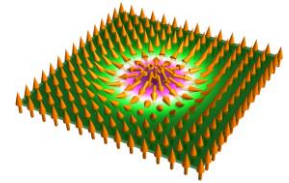
1.Topological magnetic solitons.

2. **Magnetic vortices**. Curvature effects in vortices dynamics.
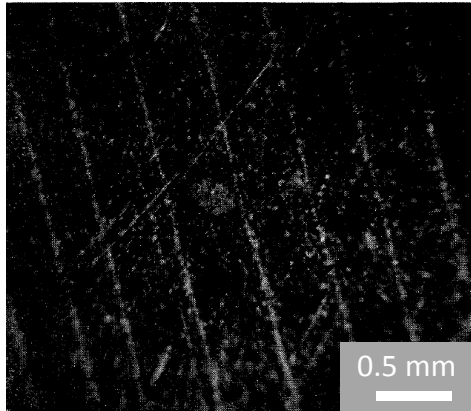
3. Curvature enters **chirality** into the game.

4. **Magnetic skyrmions**. Curvature effects in skyrmion statics and dynamics.

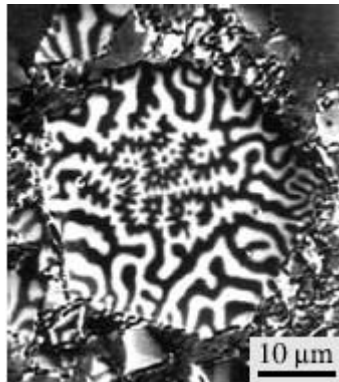5. General curvilinear approach and further perspectives.
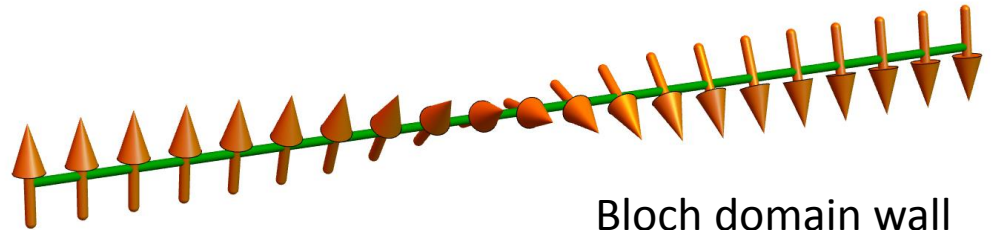
Domain wall – the simplest example of a topological soliton

Ni surface covered by suspension of Fe_2O_3 nanoparticles.
[F. Bitter, Phys. Rev. **38**, 1903 (1931)]
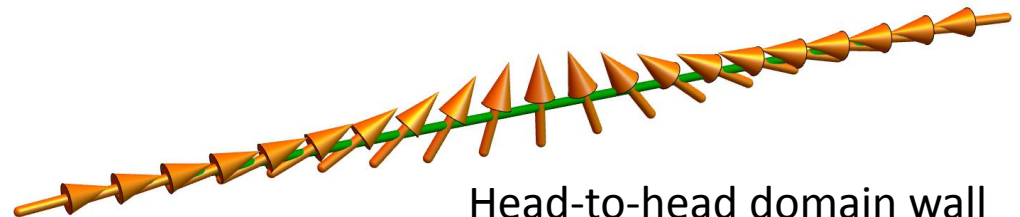

Sm_2Fe_{17}, Kerr microscopy.
[J. Zawadzki, P. A.P. Wendhausen,
B. Gebel, et al., J. Appl. Phys, **76**, 6717 (1994)]

Bloch domain wall

Neel domain wall
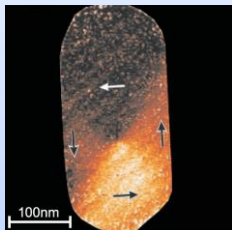
Head-to-head domain wall

Topological solitons in higher dimensions

Magnetic vortex

Minimal model:
- Isotropic exchange.
- Easy-plane anisotr.

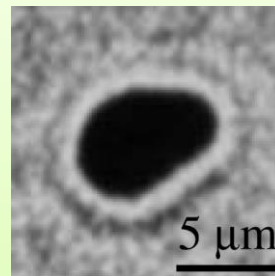→ Fe, XPEEM, [A. Wachowiak, et al., Science **298**, 577 (2002)]

[E. Feldkeller, H. Thomas, Phys. kondens. Materie **4**, 8-14 (1965)] -- first theoretical prediction.

[T. Shinjo, T. Okuno, R. Hassdorf, et al., Science **289**, 930 (2000)] -- first observation.

Magnetic bubble

Minimal model:
- Isotropic exchange.
- Magnetic field.
- Perpendicular easy-axis anisotropy.
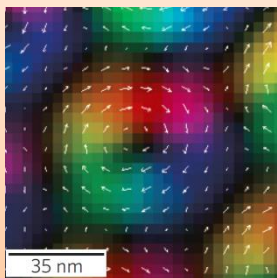- Dipole-dipole interaction

Garnet, Faraday microscopy,
[A. Hubert, R .Schäfer, Magnetic domains, Springer, 1998]

Magnetic skyrmion

Minimal model:
- Isotropic exchange.
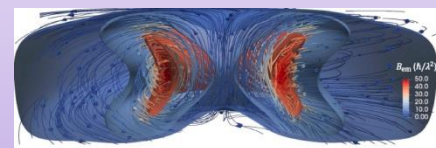- Perpendicular magnetic field /easy-axis anisotropy.
- Dzyaloshinskii-Moriya interaction.

FeGe, Lorentz TEM,
[X.Z. Yu, N. Kanazawa, Y. Onose, Nature Materials, **10**, 106, (2011)]

Hopfions
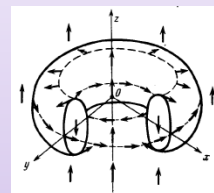
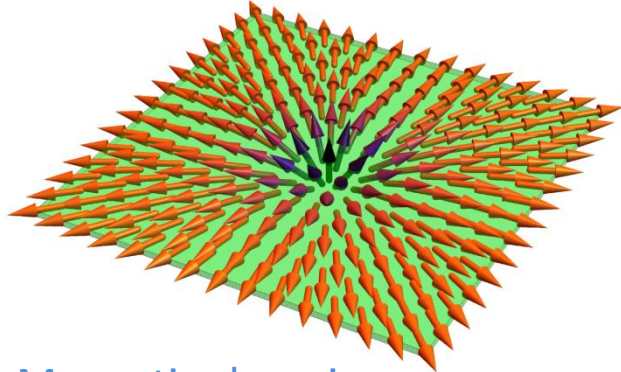[I.E. Dzyaloshinskii, B.A. Ivanov, JETP Lett., **29**, 592, 1979]

[J.-S.B. Tai, I.I. Smalyukh, PRL, **121**, 187201 (2018)]

2D topological solitons: magnetic vortices vs skyrmions.

Magnetic vortex

Minimal model:
isotropic exchange+ easy-plane anisotropy

Mapping to the unit sphere

$Q = \dfrac{1}{2}$
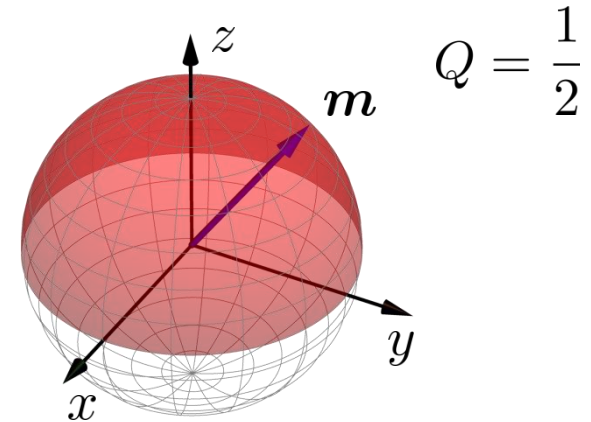
Magnetic skyrmion

Minimal model:
isotropic exchange+
easy-axis perp. anisotropy (magn. field)+
Dzyaloshinskii-Moriya interaction

$$Q = \frac{1}{4\pi} \iint \boldsymbol{m} \cdot [\partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m}] \mathrm{d}x\mathrm{d}y$$

Mapping to the unit sphere

$Q = 1$
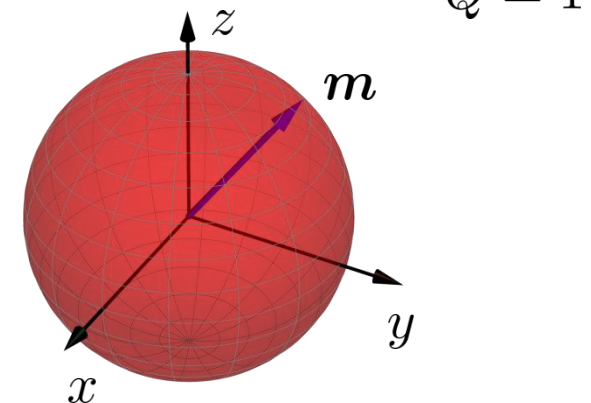
VRAM – Vortex Random Access Memory

Main characteristics:

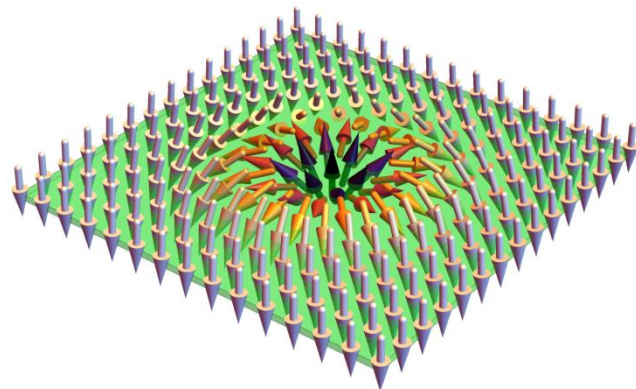- Non-volatility

- Speed of operation (~1ns, comparable with modern DRAM)

- Data density (comparable with modern DRAM)

- Energy consumption is 99% less than in the DRAM

[Young-Sang Yu et al., Appl. Phys. Lett. **98**, 052507 (2011)]

Mechanism of the vortex polarity switching

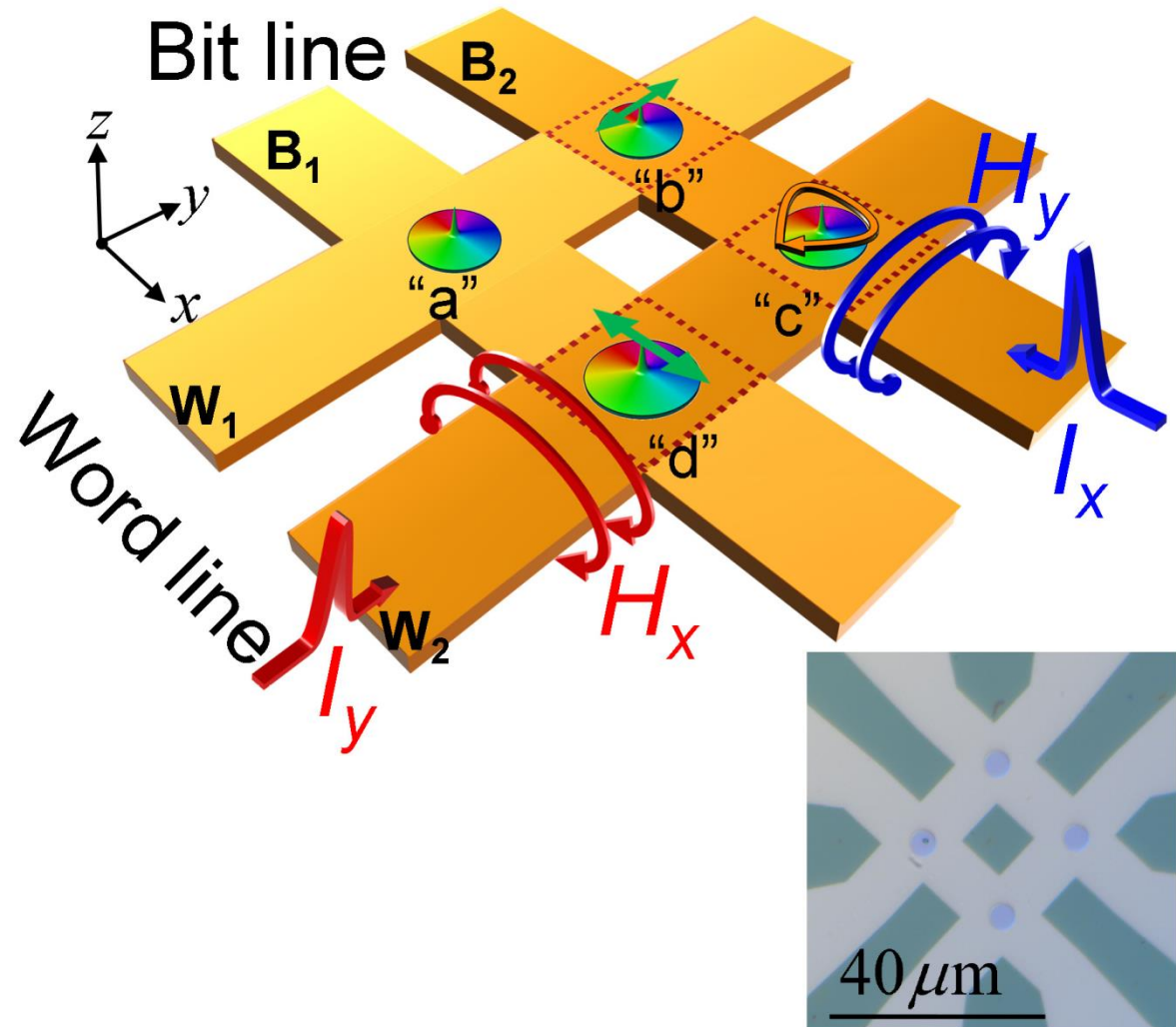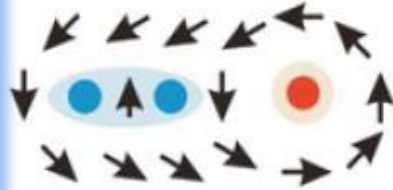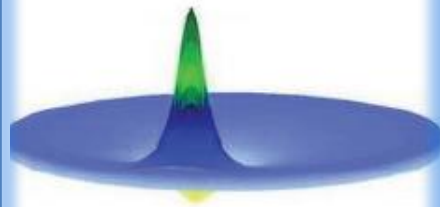Vortex moves under external driving (magn. field, spin-current)

Dip appears next to the vortex core. Vortex-artivortex pair is born from the dip. Polarities on new particles are opposite to polarity of the initial vortex.
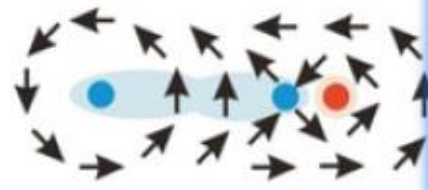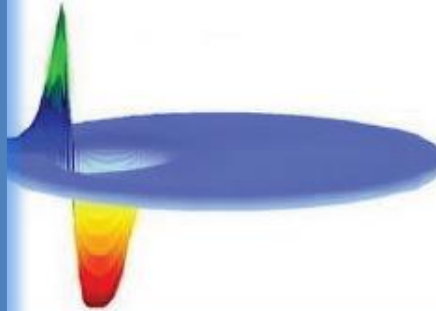
New antivortex annihilates with the initial vortex.
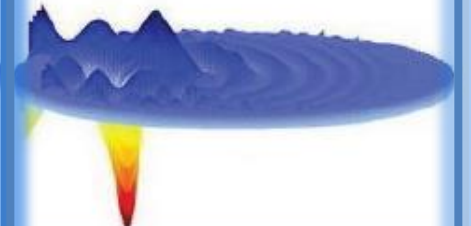
[B. van Waeyenberge, A. Puzic, H. Stoll, et al., Nature, **444**, 461, (2006)]

[R. Hertel, S. Gliga, M. Fähnle, C. Schneider, PRL, **98**, 117201 (2007)]

[D. Sheka, Yu. Gaididei, F. Mertens, APL, **91**, 082509 (2007)]

[K. Guslienko, K.-S. Lee, S.-K. Kim, PRL, **100**, 027203 (2008)]   [K.-S. Lee, S.-K. Kim, Y.-S. Yu et al., PRL, **100**, 027203 (2008)]

Chirality symmetry breaking in vortex polarity switching
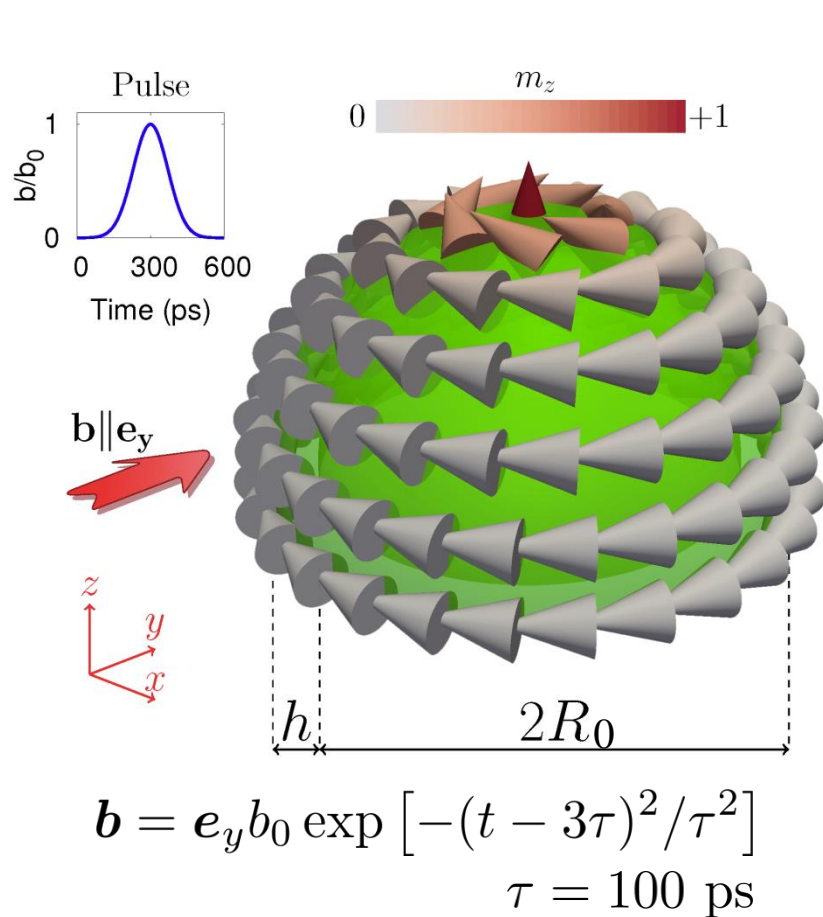
Volume and thickness are constant

$h = 10$ nm

$R = R_0/\sqrt{1 - \cos\theta}$

$\boldsymbol{b} = \boldsymbol{e}_y b_0 \exp\left[-(t - 3\tau)^2/\tau^2\right]$

$\tau = 100$ ps

Pulse

b/b_0

Time (ps)

m_z

$\mathbf{b}\|\mathbf{e_y}$

$2R_0$

Critical amplitude b_c (mT)

Inner radius R (nm)

p=+1, c= -1
p= -1, c= -1
p=+1, c=+1
p= -1, c=+1

$c = +1$

$c = -1$
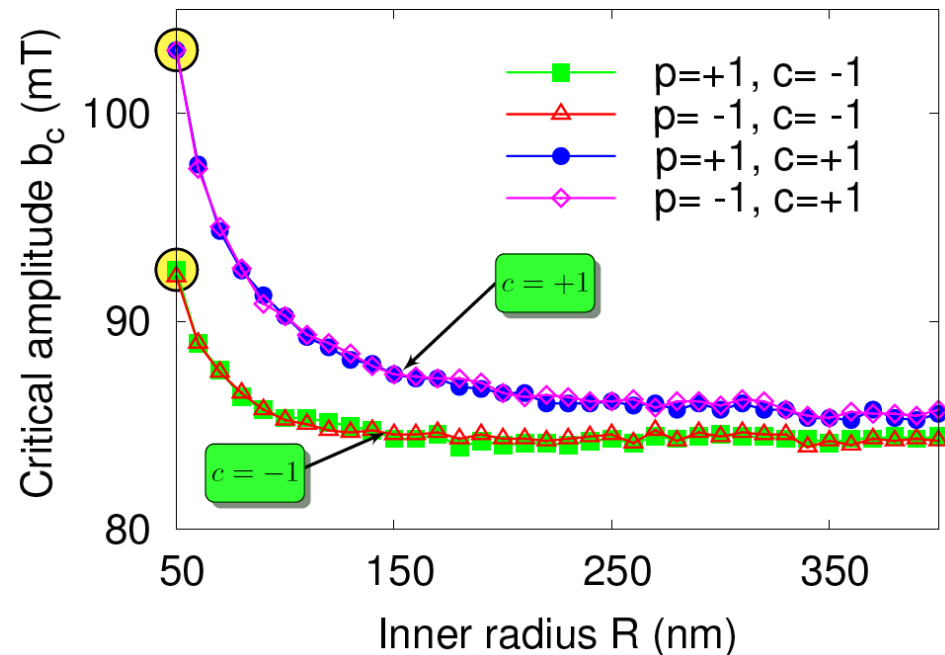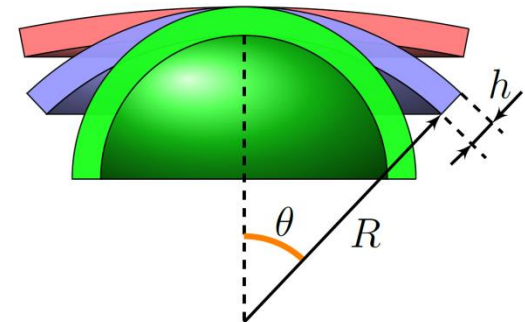
[M. Sloika, V. Kravchuk, D. Sheka, Yu. Gaididei , Appl. Phys. Lett., **104**, 252403 (2014)]

Controllable chirality switching of a spherical cap

$$c(t) = \frac{1}{V} \int m_\chi(\boldsymbol{r}, t)\,\mathrm{d}\boldsymbol{r}$$
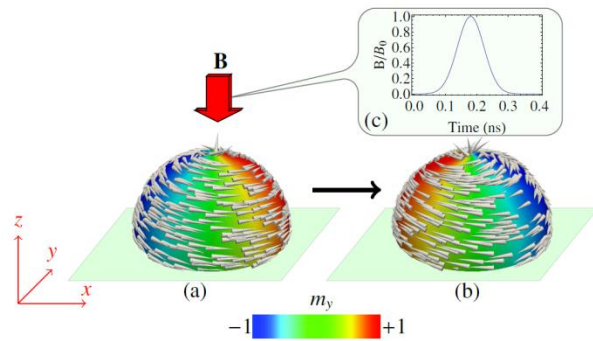
Pulse ampl. = 300mT

Sample: Permalloy,
h = 10 nm, R_{sphere} = 100 nm, R_{disk} = 141 nm.

Gaussian pulse: amplitude 300 mT, width 20 ps.

[K. Yershov, V. Kravchuk, D. Sheka, Yu. Gaididei, J. Appl. Phys., **117**, 083908 (2015)]

Controllable chirality switching of a spherical cap
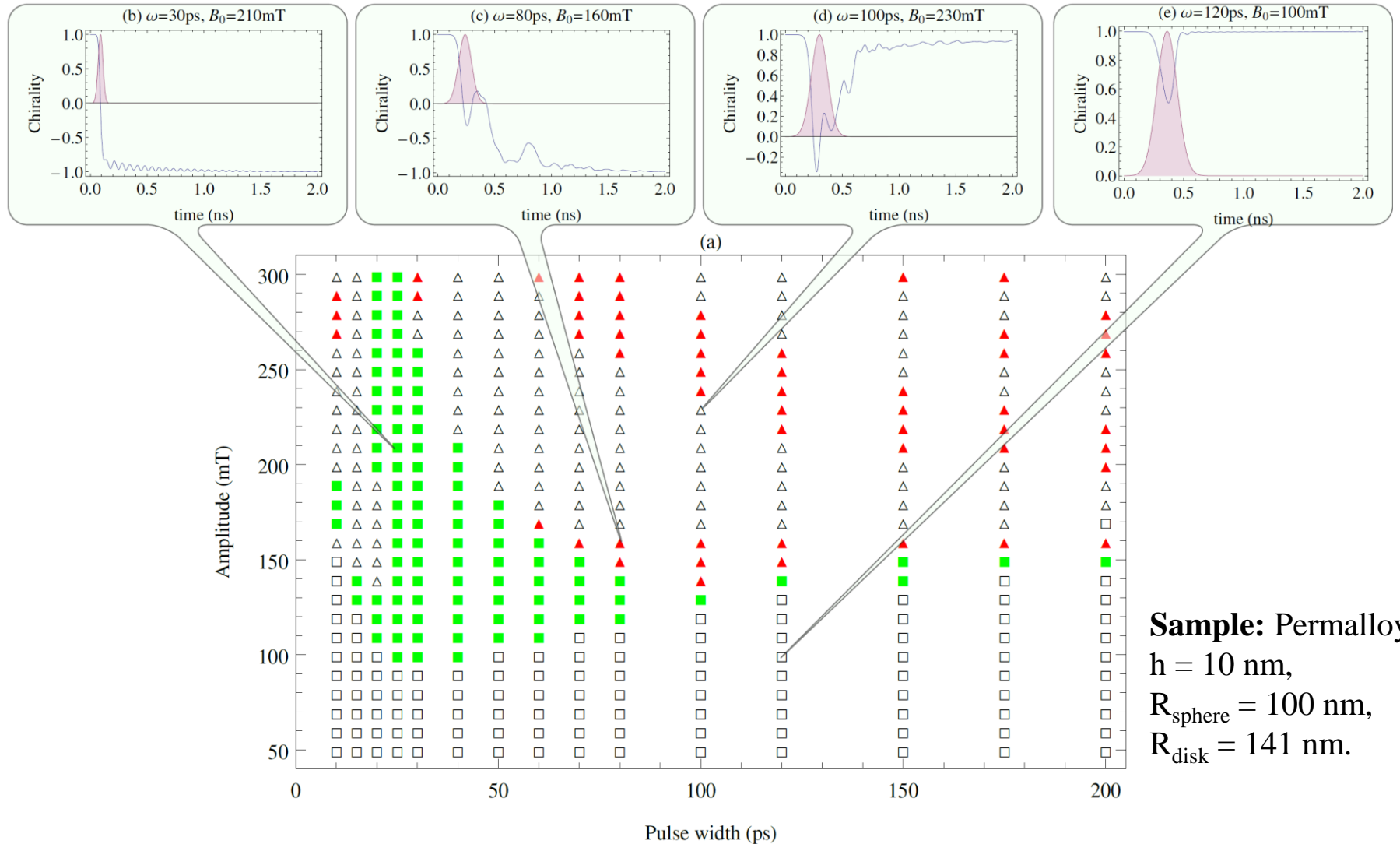
Sample: Permalloy,
h = 10 nm,
R_{sphere} = 100 nm,
R_{disk} = 141 nm.

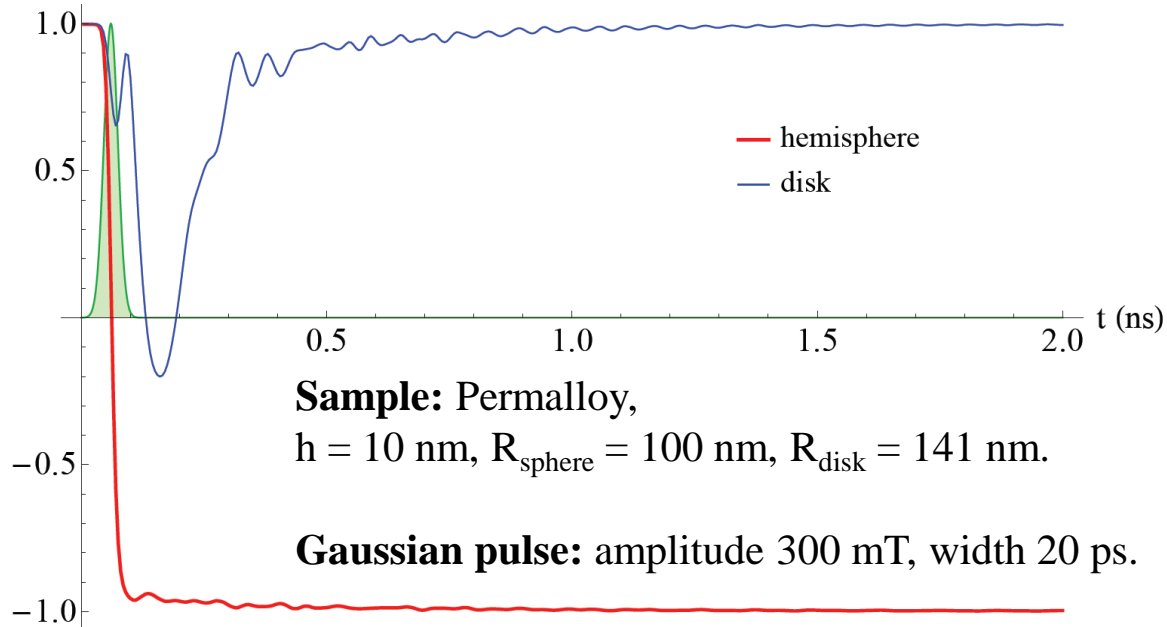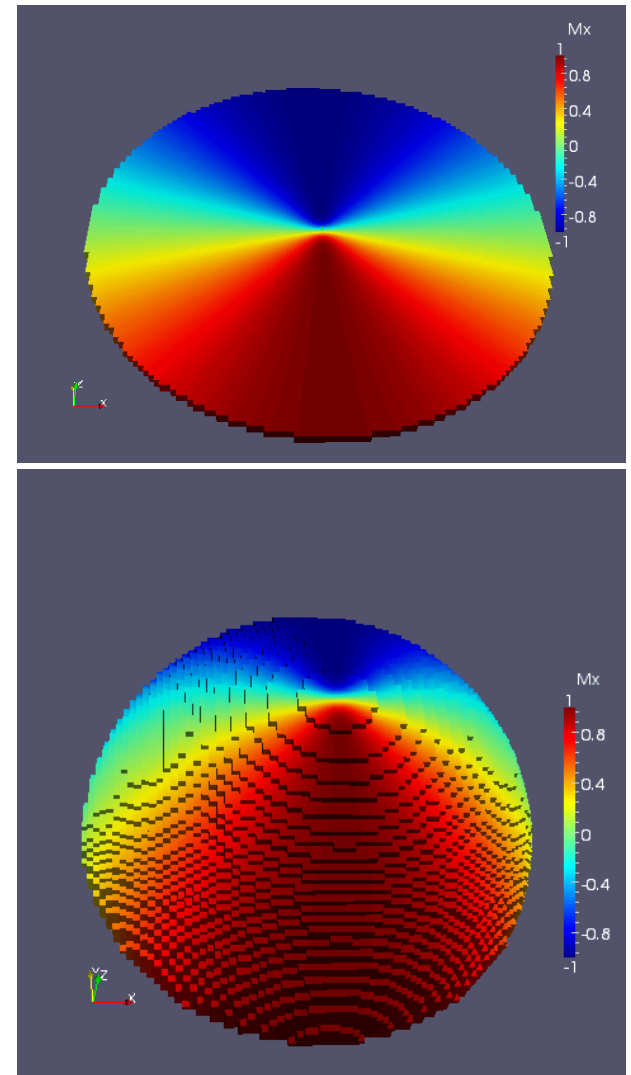[K. Yershov, V. Kravchuk, D. Sheka, Yu. Gaididei, J. Appl. Phys., **117**, 083908 (2015)]

Curvature couples polarity and chirality of the vortex

Being of small amplitude some effects can be spatially nonlocal.

Vortex on a planar disk
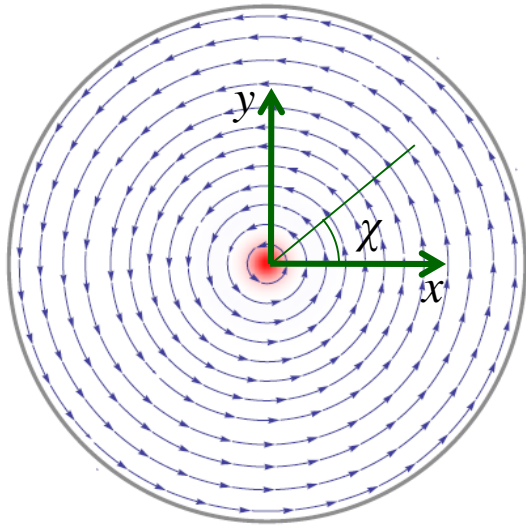
$$\varphi = \chi + \mathfrak{C}\frac{\pi}{2}$$

$$\mathfrak{C} = \pm 1 \ \text{- chirality,}$$

$$\tan\varphi = m_y/m_x$$

Vortex on a sphere
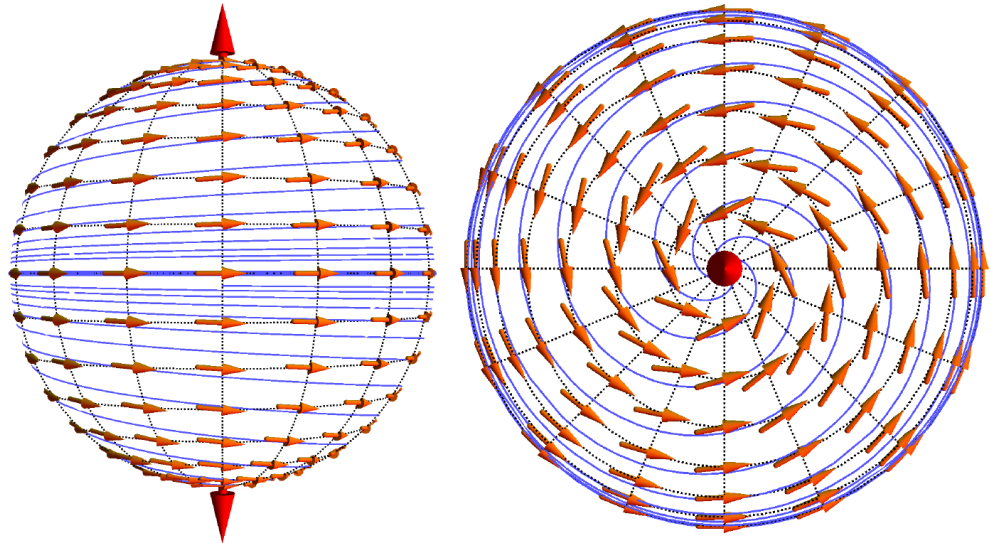
$$\varphi = \chi + \mathfrak{C}\frac{\pi}{2}\left(1 - p\frac{\ell}{\mathcal{R}}\xi\ln\tan\frac{\vartheta}{2}\right)$$

p - polarity,

$\mathcal{R}$ - sphere radius,

ϑ - polar angle

Can the nonlocal magnetization deformation modify the vortex-vortex coupling on a sphere?

[V.P. Kravchuk, D. Sheka, R. Streubel, D. Makarov, et al., Phys. Rev. B, **85**, 144433 (2012)]

Curvature induced chirality symmetry breaking in moving vortex DW
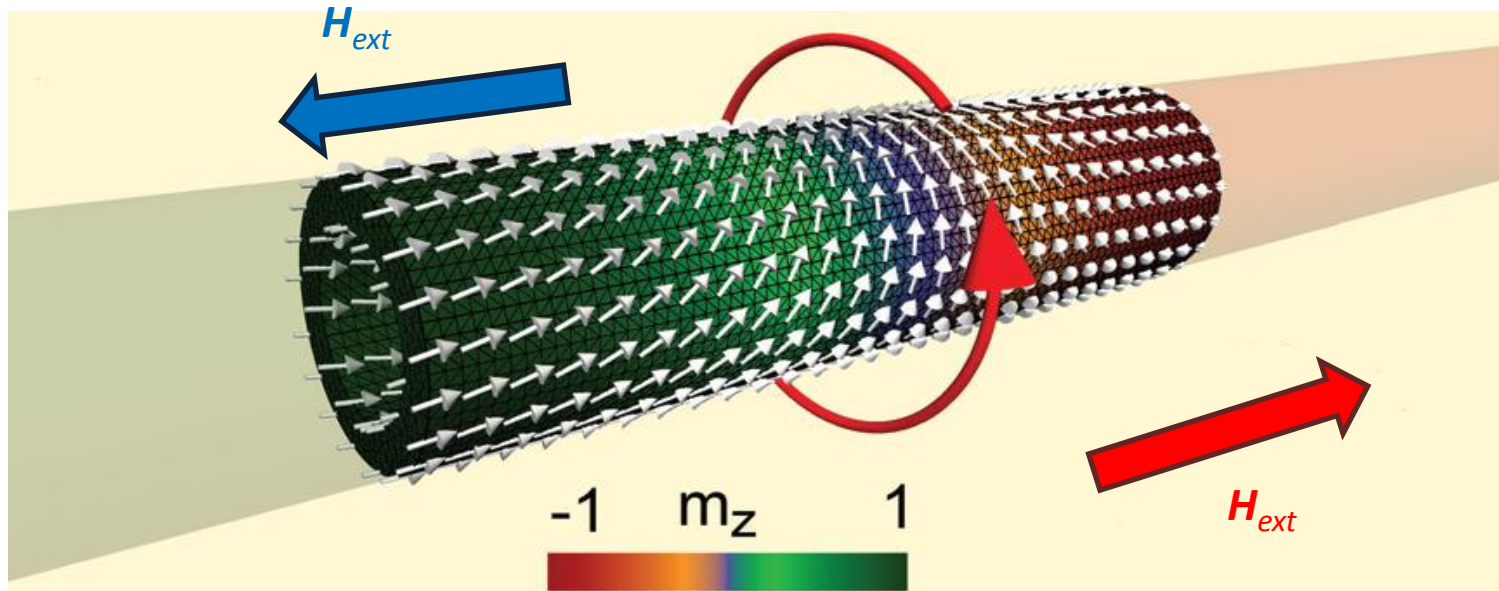
Permalloy,
R=60 nm,
h=10 nm

[Appl. Phys. Lett., **100**, 252401, (2012)]
R. Hertel group
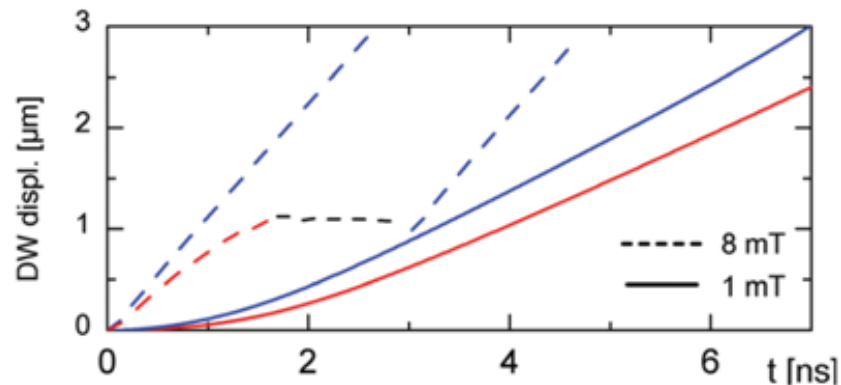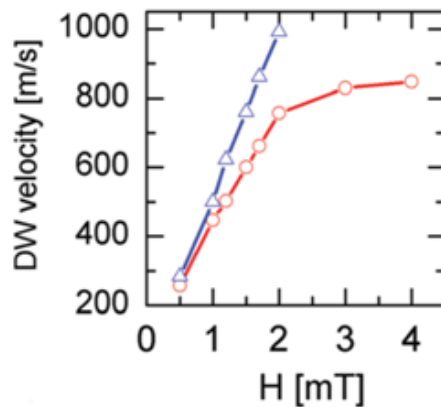
[Appl. Phys. Lett., **100**, 072407, (2012)]
P. Landeros group

Domain walls on a Möbius strip

Permalloy

$R = 100\,\mathrm{nm}$
$w = 80\,\mathrm{nm}$

$Q = K/(2\pi M_s^2)$

[O. Pylypovskyi, V. Kravchuk, D. Sheka, D. Makarov,
O. Schmidt, Yu. Gaididei, Phys. Rev. Lett. **114**, 197204 (2015)]
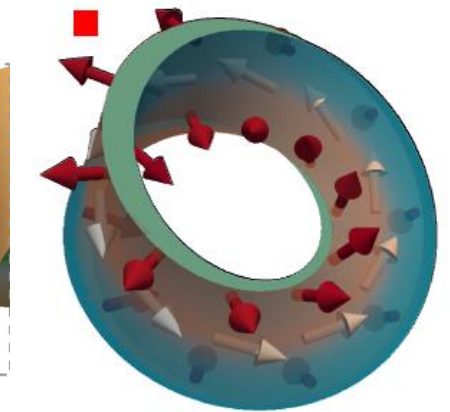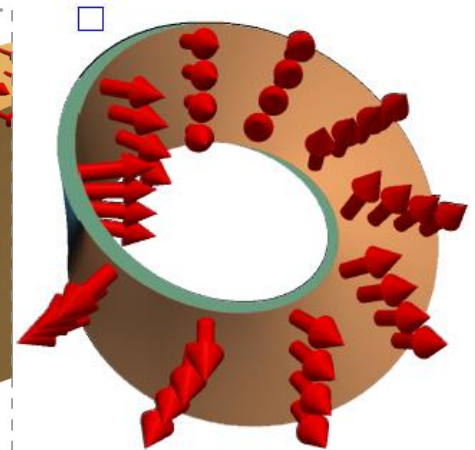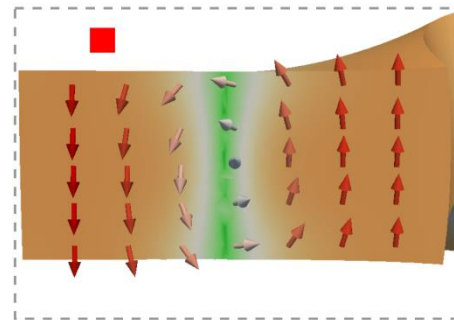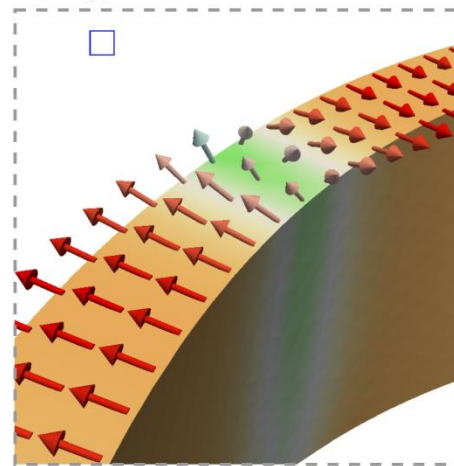
Longitudinal domain wall on a Möbius strip

$$x + iy = \left(R + \xi \cos \frac{\chi}{2} \right) e^{i\chi}, \qquad 0 \le \chi < 2\pi,$$

$$z = \mathcal{C} \, \xi \sin \frac{\chi}{2}, \qquad -\frac{w}{2} \le \xi \le \frac{w}{2}$$

$$\boldsymbol{m} = \sin\theta \cos\phi \, \boldsymbol{e}_\chi + \sin\theta \sin\phi \, \boldsymbol{e}_\xi + \cos\theta \, \boldsymbol{n}$$
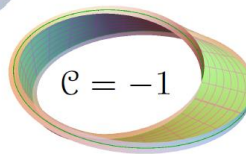
$$\theta^\ell = 2 \arctan e^{p\frac{\xi}{d}}, \qquad \phi^\ell = \pi \frac{\mathfrak{c} + p}{2}$$
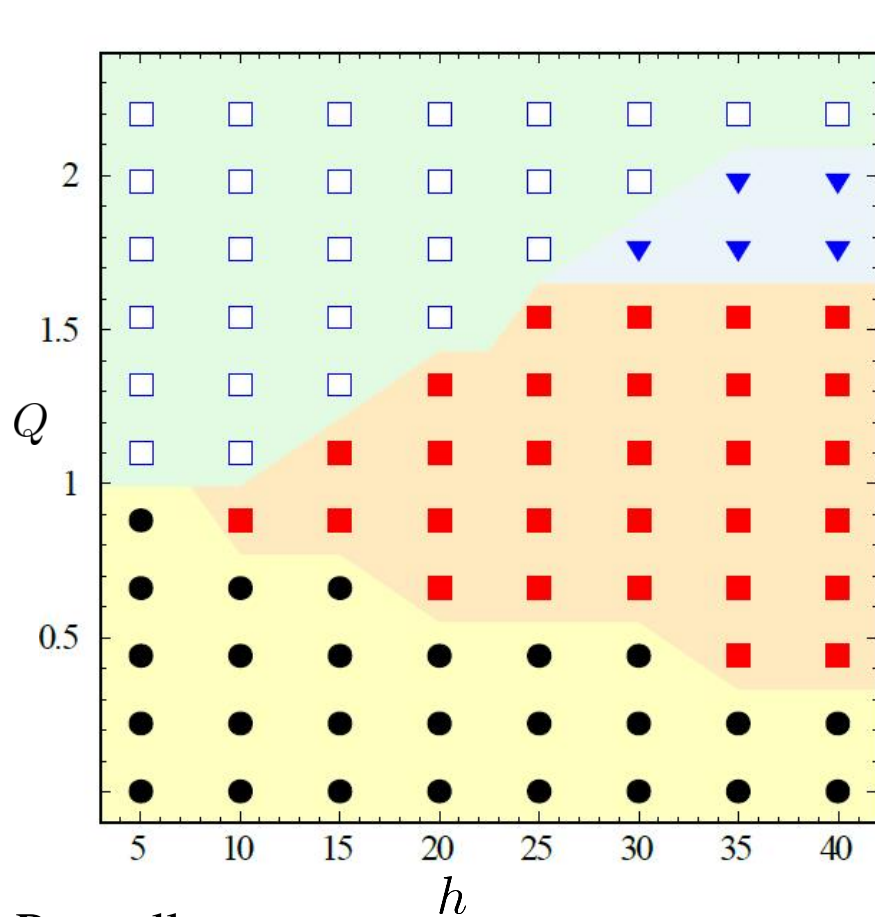
$$\frac{E^\ell}{4\pi A h} \approx \frac{R}{d} - \frac{\pi}{2}\mathcal{C}\mathfrak{c} + \varepsilon_0 \frac{d}{R} + \frac{dR}{\ell^2} + \text{const}$$

$$\varepsilon_0 = 1/4 + \pi^2/96$$
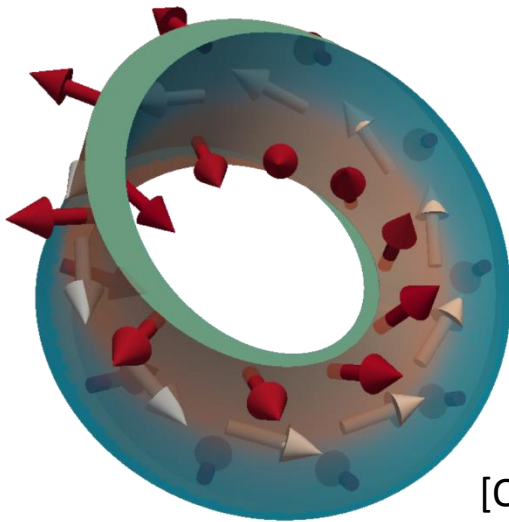
[O. Pylypovskyi, V. Kravchuk, D. Sheka, D. Makarov, O. Schmidt, Yu.Gaididei, Phys. Rev. Lett. **114**, 197204 (2015)]

Magnetic skyrmion is a topologically stable localized excitation in perpendicularly magnetized thin films

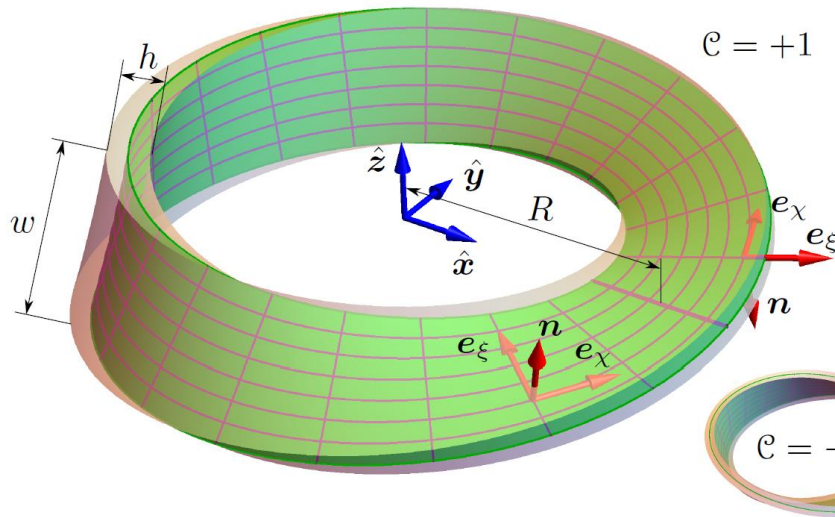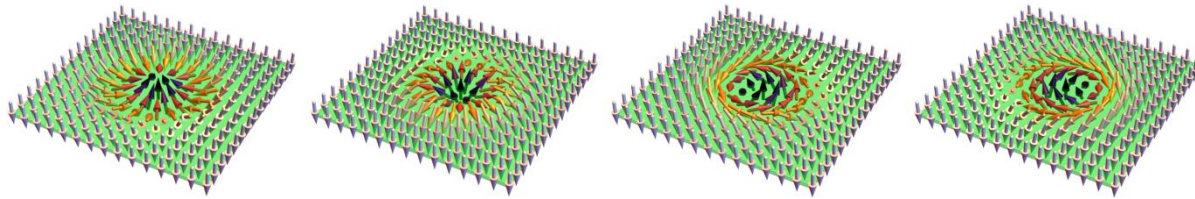$$Q = \frac{1}{4\pi} \iint \boldsymbol{m} \cdot [\partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m}] \mathrm{d}x\mathrm{d}y = \pm 1$$

Experimental observations and applications for racetrack memory and logic devices
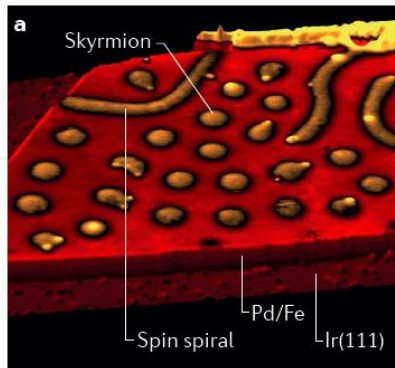
[R. Wiesendanger, Nature Rev. Mater., **1**, 16044 (2016)]
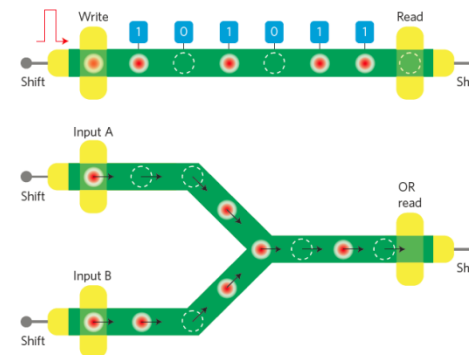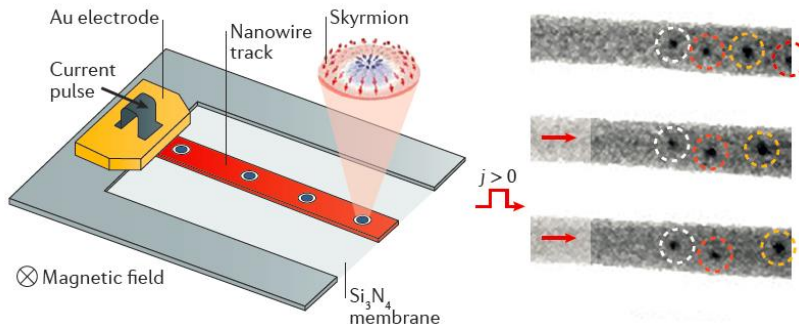
[S. Krause, R. Wiesendanger, Nature. Mater, **15**, 493 (2016)]
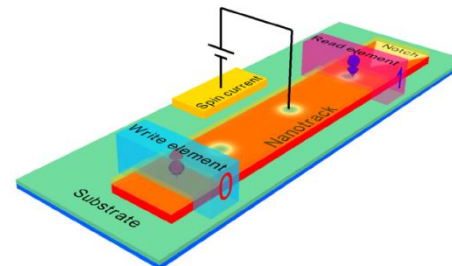
[X. Zhang, et al., Scientific Reports 5 : 7643 (2015)

Skyrmion on a planar film. Minimal model.

$$E = L \int \left\{ A\,\mathscr{E}_{\mathrm{ex}} + K\left[1 - (\boldsymbol{m} \cdot \boldsymbol{n})^2\right] + D\,\mathscr{E}_{\mathrm{D}} \right\} \mathrm{d}S$$ - uniaxial magnet with interfacial DMI

$$\mathscr{E}_{\mathrm{ex}} = \partial_i \boldsymbol{m} \cdot \partial_i \boldsymbol{m}$$

$$\mathscr{E}_{\mathrm{D}} = m_n \boldsymbol{\nabla} \cdot \boldsymbol{m} - (\boldsymbol{m} \cdot \boldsymbol{\nabla}) m_n$$

$\mathcal{C} = +1$

$\mathcal{C} = -1$
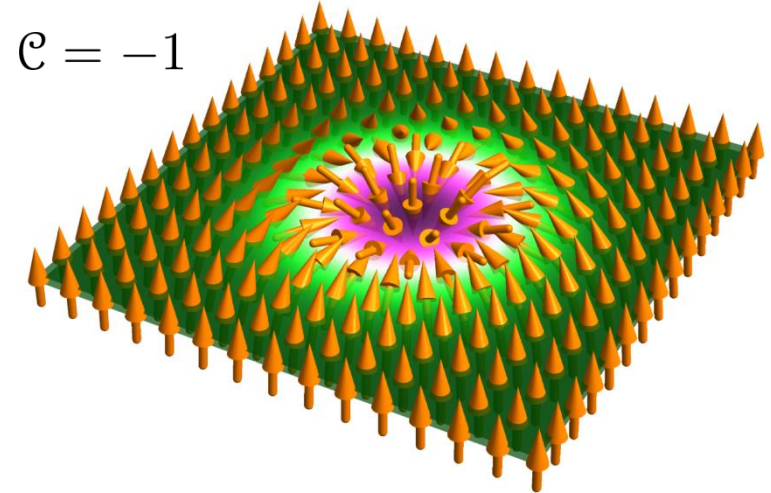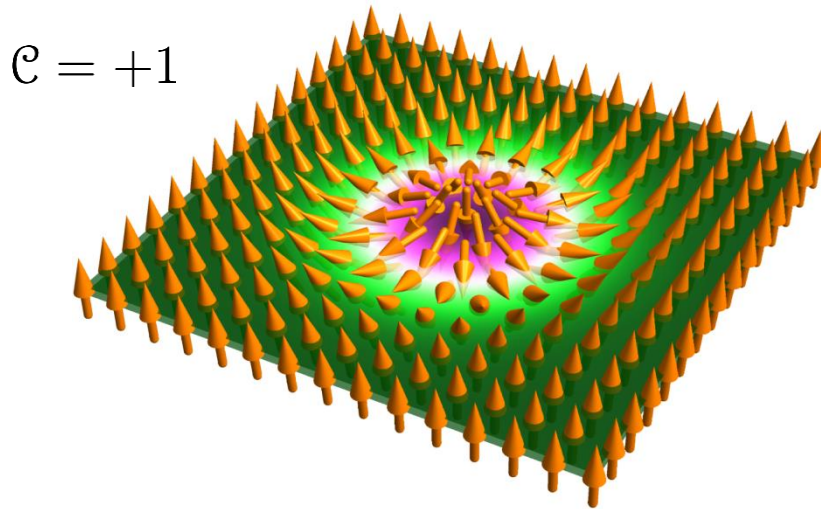
Estimations for small radius skyrmion:

$$E_{\mathrm{ex}} \approx 8\pi A h, \quad E_{\mathrm{a}} \propto K R_s^2, \quad E_{\mathrm{D}} \propto -\mathcal{C} D R_s,$$

$$R_s \propto |D|/K,$$

Exchange driven curvature effects.

$$\boldsymbol{n} = [\boldsymbol{e}_1 \times \boldsymbol{e}_2]$$
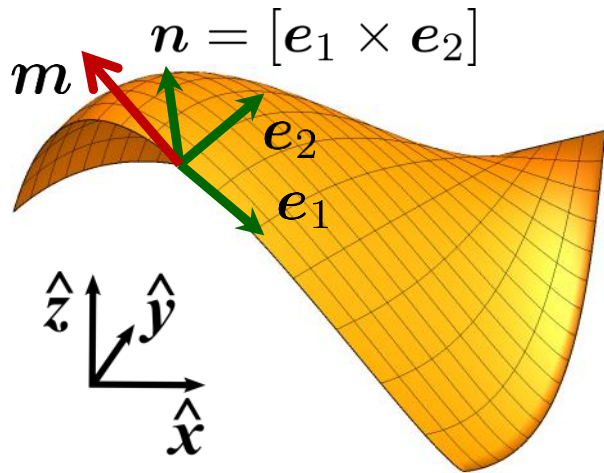
$$\mathscr{E}_{ex} = \partial_i m_j \partial_i m_j, \qquad (i, j = x, y, z)$$

$$\mathscr{E}_{ex} = \bar{\partial}_\alpha m_\beta \bar{\partial}_\alpha m_\beta + \bar{\partial}_\alpha m_n \bar{\partial}_\alpha m_n \quad \Rightarrow \quad \textbf{common exchange}$$

$$+ 2 h_{\alpha\beta} \mathcal{L}_{\beta n}^{(\alpha)} \quad \Rightarrow \quad \textbf{DMI}$$

$$+ h_{\alpha\gamma} h_{\gamma\beta} m_\alpha m_\beta + (\mathcal{H}^2 - 2\mathcal{K}) m_n^2 \quad \Rightarrow \quad \textbf{Anisotropy}$$

$$(\alpha, \beta, \gamma = 1, 2)$$

$m_\alpha = (\boldsymbol{m} \cdot \boldsymbol{e}_\alpha)$ - tangential components

$m_n = (\boldsymbol{m} \cdot \boldsymbol{n})$ - normal component

$\boldsymbol{\nabla} \equiv (g_{\alpha\alpha})^{-1/2} \boldsymbol{e}_\alpha \partial_\alpha$ - surface gradient

$||g_{\alpha\beta}||$ - metric tensor (**diagonal**)

$\bar{\partial}_\alpha m_\beta = \nabla_\alpha m_\beta + \epsilon_{\beta\gamma} \Omega_\alpha m_\gamma$ -covariant derivative

$\bar{\partial}_\alpha m_n = \nabla_\alpha m_n$

$\Omega_\gamma = \dfrac{1}{2} \epsilon_{\alpha\beta} \boldsymbol{e}_\alpha \cdot \nabla_\gamma \boldsymbol{e}_\beta$ - spin connection

Lifshitz invariants

$$\mathcal{L}_{\beta n}^{(\alpha)} = m_\beta \bar{\partial}_\alpha m_n - m_n \bar{\partial}_\alpha m_\beta$$

$h_{\alpha\beta} = \boldsymbol{e}_\beta \cdot (\boldsymbol{e}_\alpha \cdot \boldsymbol{\nabla}) \boldsymbol{n}$ - Weingarten map

Mean curvature $\quad \mathcal{H} = \mathrm{tr}||h_{\alpha\beta}|| = k_1 + k_2$

Gaußian curvature $\quad \mathcal{K} = \det||h_{\alpha\beta}|| = k_1 k_2$

A "Tool box" for micromagnetics of curvilinear films

Exchange energy

$$\mathcal{E}_{ex} = \partial_i m_j \partial_i m_j = [\boldsymbol{\nabla}\theta - \boldsymbol{\Gamma}]^2 + [\sin\theta(\boldsymbol{\nabla}\phi - \boldsymbol{\Omega}) - \cos\theta\partial_\phi\boldsymbol{\Gamma}]^2$$

$$m_n = \cos\theta$$
$$m_1 + im_2 = \sin\theta e^{i\phi}$$

$$\boldsymbol{\Gamma} = ||h_{\alpha\beta}|| \cdot \boldsymbol{\varepsilon}, \qquad \boldsymbol{\varepsilon} = \cos\phi\, \boldsymbol{e}_1 + \sin\phi\, \boldsymbol{e}_2$$

[Yu. Gaididei, V. Kravchuk, D. Sheka, *Phys. Rev. Lett*. **112**, 257203 (2014)]

DMI

$$\mathcal{E}_{\mathrm{D}}^{\partial\mathrm{N}} = m_n \boldsymbol{\nabla}\cdot\boldsymbol{m} - (\boldsymbol{m}\cdot\boldsymbol{\nabla})m_n = \sin^2\theta\left[2(\boldsymbol{\nabla}\cdot\boldsymbol{\varepsilon}) + \mathcal{H}\right]$$

[V. Kravchuk, D. Sheka, A. Kakay, *et al., Phys. Rev. Lett*. **120**, 067201 (2018)]

$$\mathcal{E}_{\mathrm{D}}^{\partial\mathrm{B}} = \boldsymbol{m}\cdot[\boldsymbol{\nabla}\times\boldsymbol{m}] = \sin^2\theta\left[(2\boldsymbol{\nabla}\theta - \boldsymbol{\Gamma})\times\boldsymbol{\varepsilon}\right]$$
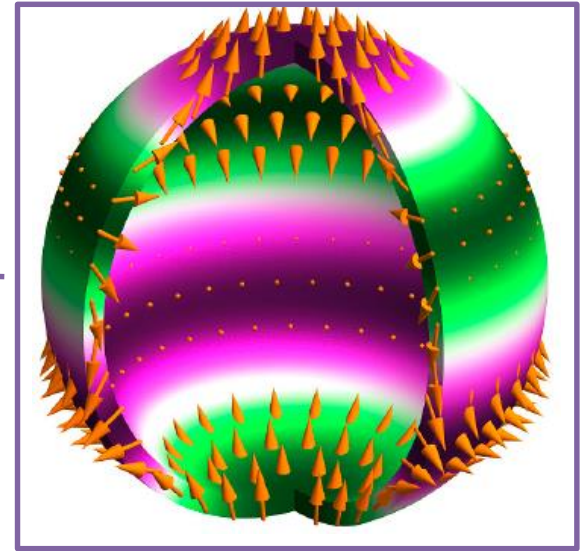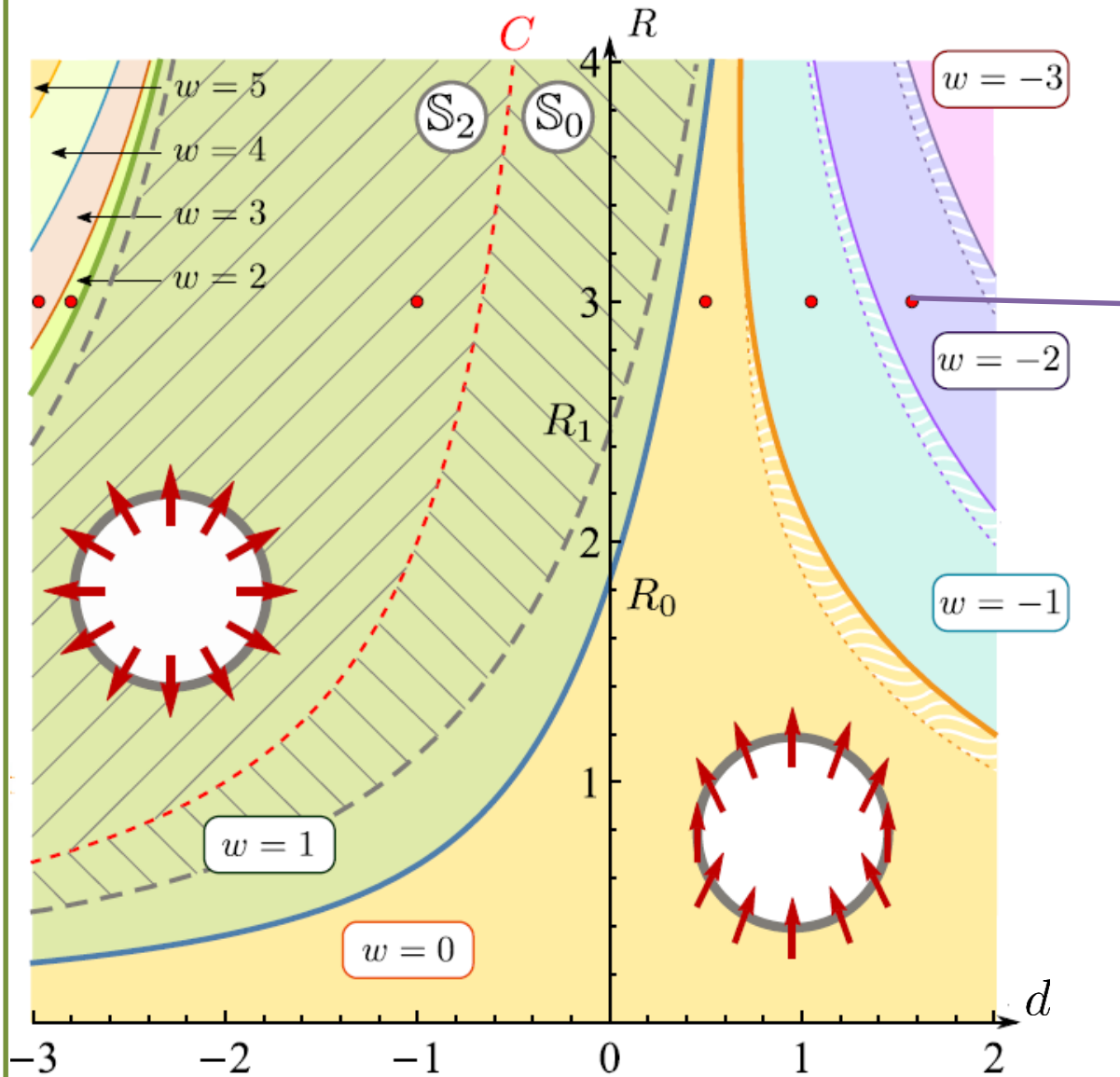
Magnetic interactions containing spatial derivatives are sources of new effective interactions on curvilinear surfaces.

Topological charge density

$$\mathcal{J} = \boldsymbol{m}\cdot[\partial_x\boldsymbol{m}\times\partial_y\boldsymbol{m}] = \sin\theta\left[(\boldsymbol{\nabla}\theta - \boldsymbol{\Gamma})\times(\boldsymbol{\nabla}\phi - \boldsymbol{\Omega})\right]_n + \cos\theta\left([\partial_\phi\boldsymbol{\Gamma}\times\boldsymbol{\nabla}\theta]_n + \mathcal{K}\right)$$

[V. Kravchuk, U. Rößler, O. Volkov, *et al., Phys. Rev. B* **94**, 144402 (2016)]

Spherical shell

C

R

$w = 5$

$w = 4$

$w = 3$

$w = 2$

$\mathbb{S}_2$ $\mathbb{S}_0$

$w = -3$

$w = -2$

R_1

$w = -1$

R_0

$w = 1$

$w = 0$

d

[V. Kravchuk, U. Rößler, O. Volkov, *et al., Phys. Rev. B* **94**, 144402 (2016)]

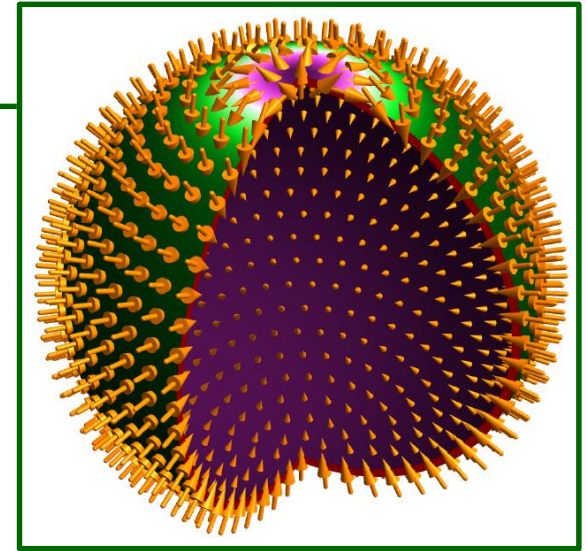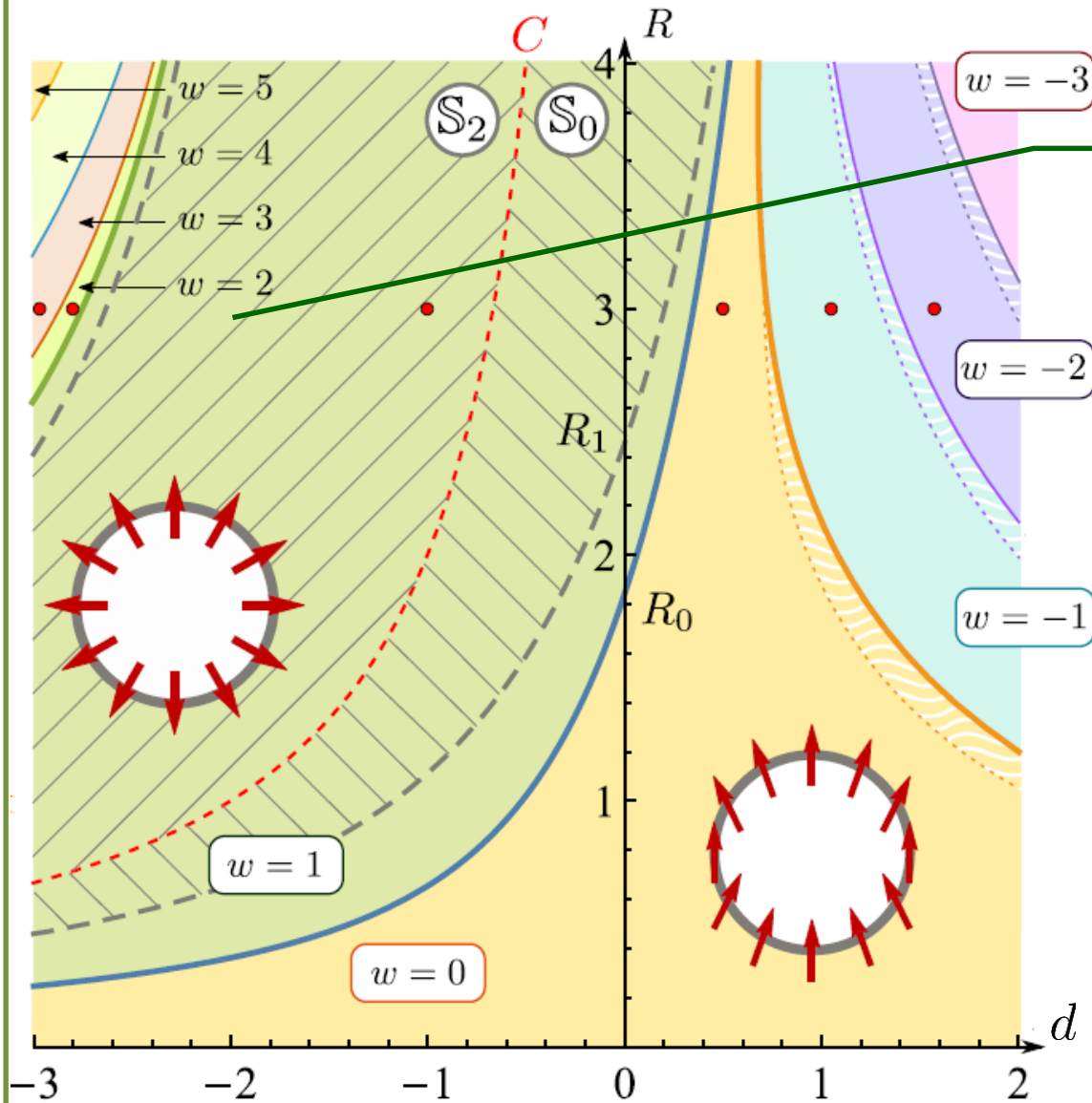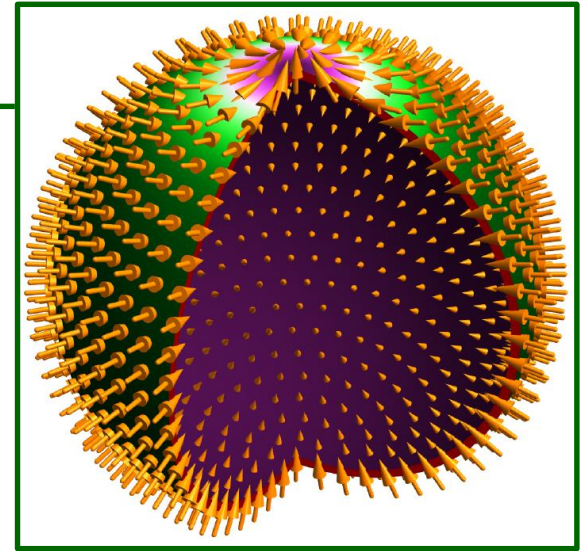Each state is doubly degenerate with respect to the transformation $m \rightarrow -m$

Spherical shell

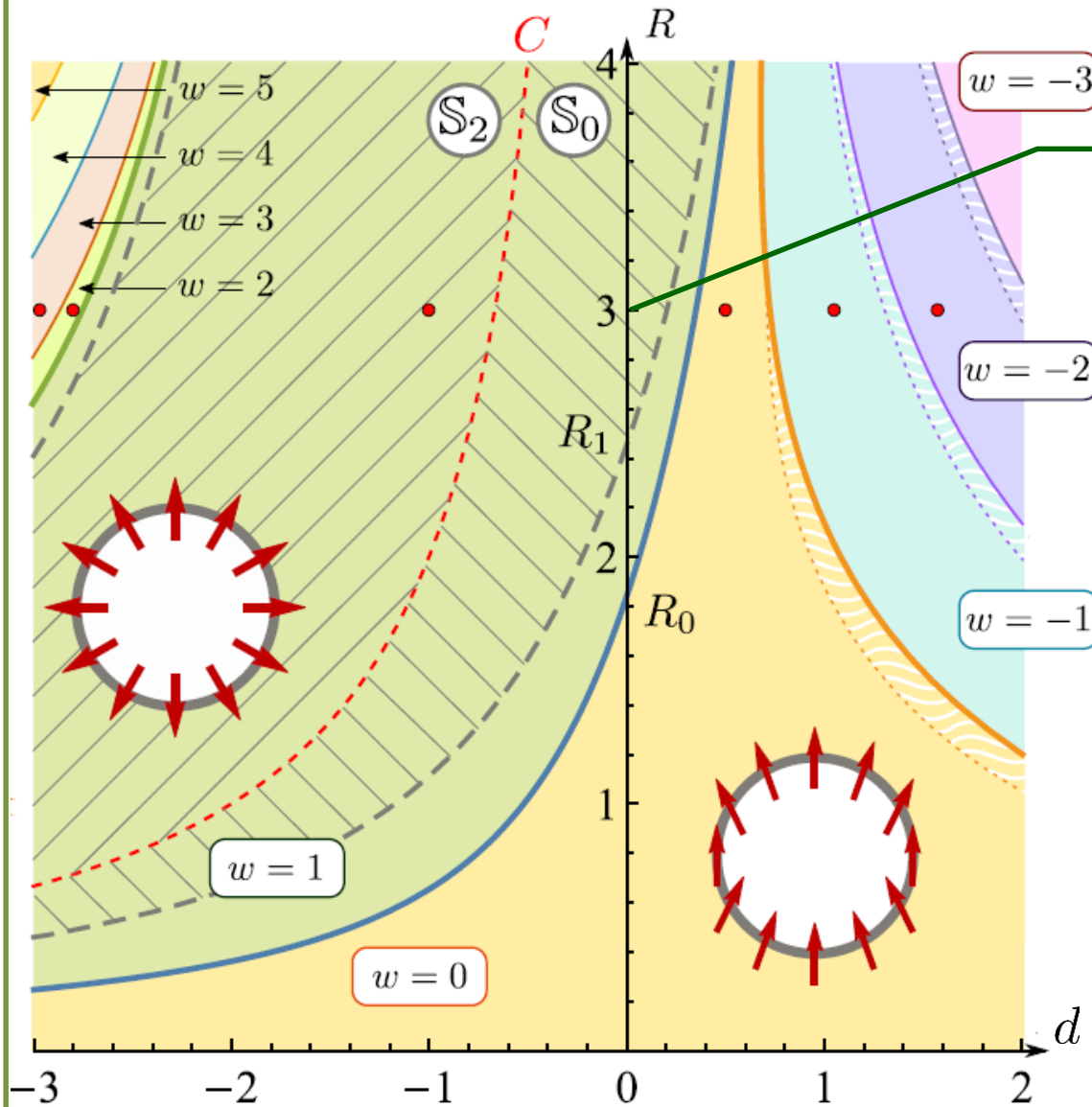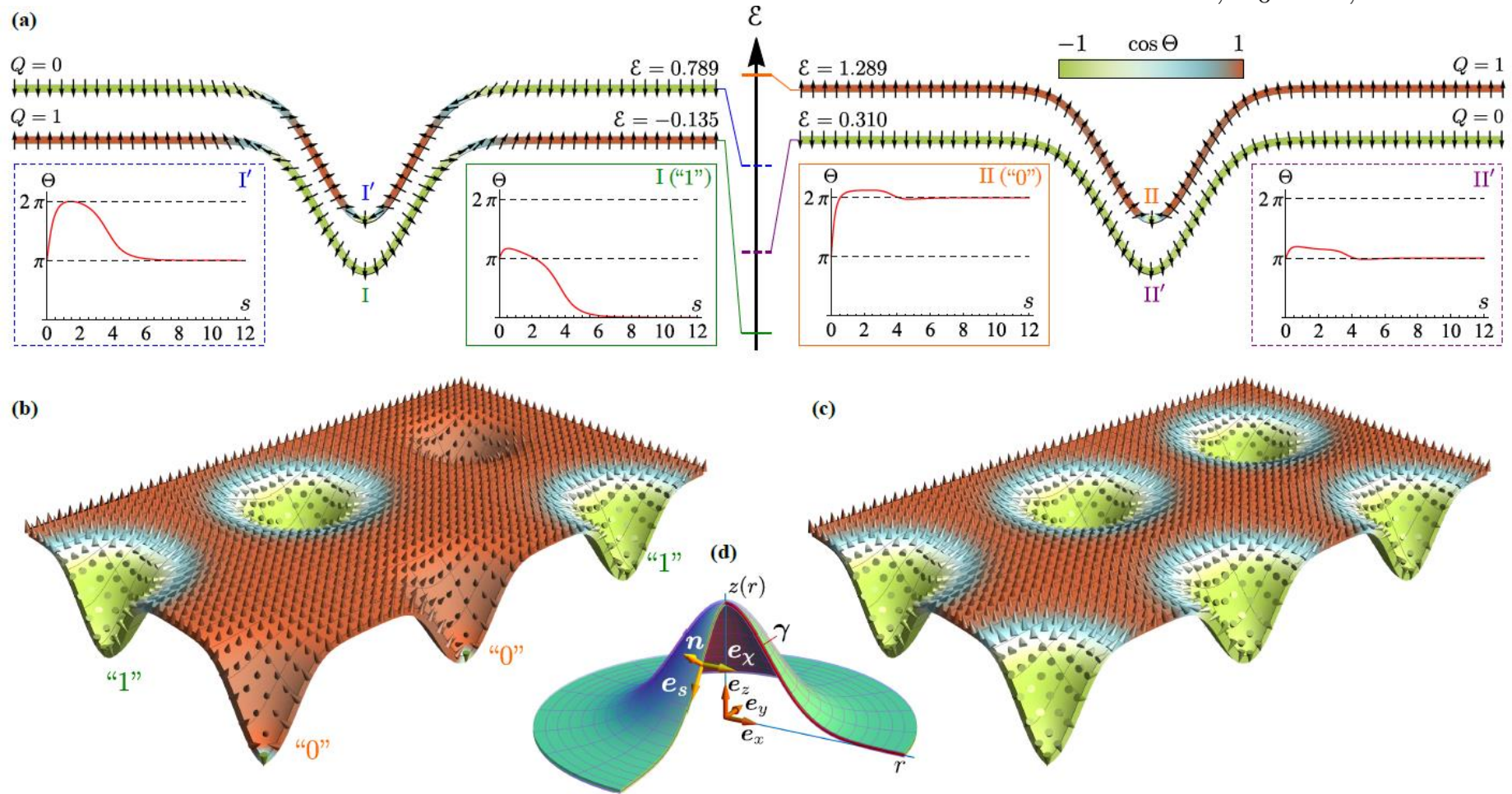Each state is doubly degenerate with respect to the transformation $m \to -m$

Spherical shell

[V. Kravchuk, U. Rößler, O. Volkov, *et al., Phys. Rev. B* **94**, 144402 (2016)]

Each state is doubly degenerate with respect to the transformation $m \to -m$

Multiplet of skyrmion states on a Gaußian bumb $z = \mathcal{A}e^{-r^2/(2r_0^2)}$
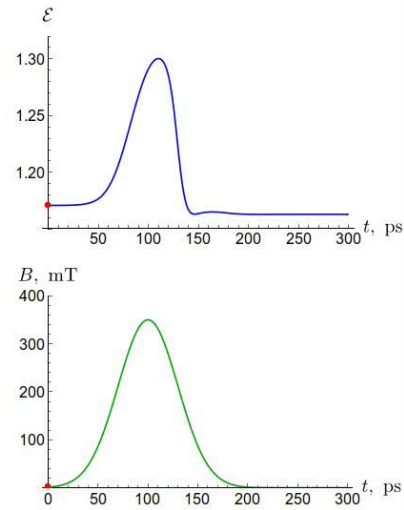
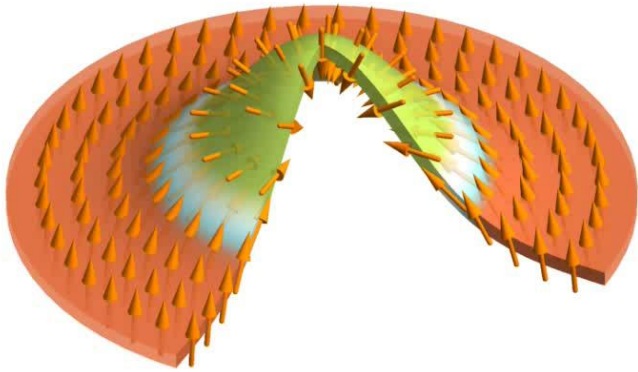$$\mathcal{A} = -3, \ r_0 = 1, \quad d = 1$$

[V. Kravchuk, D. Sheka, A. Kakay, *et al., Phys. Rev. Lett.* **120**, 067201 (2018)]

Switching between skyrmion states
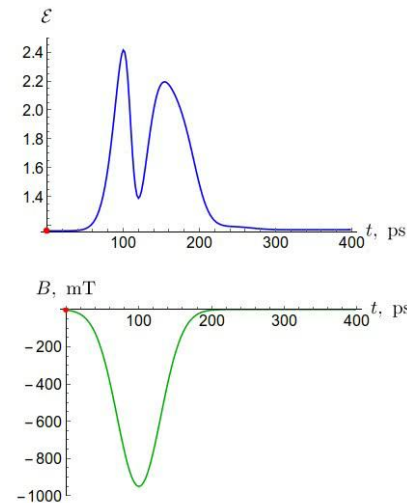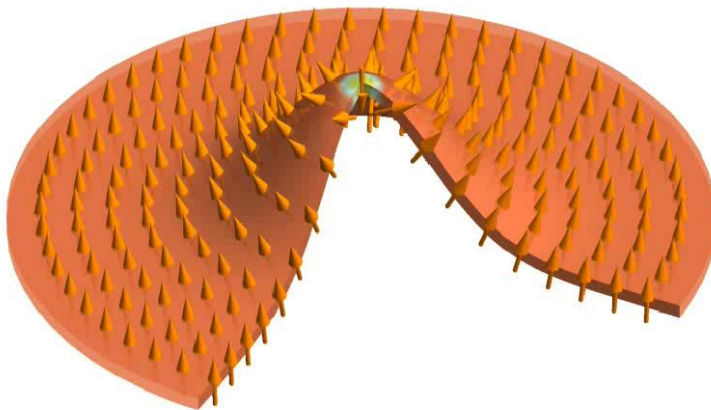
$$z = \mathcal{A}e^{-r^2/(2r_0^2)}$$

Parameters of Pl/Co/AlOx layer structures:

$$A = 1.6 \times 10^{-11} \text{ J/m}$$

$$M_{\mathrm{s}} = 1.38 \text{ T}$$

$$K = 1.3 \times 10^6 \text{ J/m}^3$$

$$K_{\mathrm{eff}} = K - 2\pi M_s^2$$

$$\ell = \sqrt{A/K_{\mathrm{eff}}} = 5.6 \text{ nm}$$

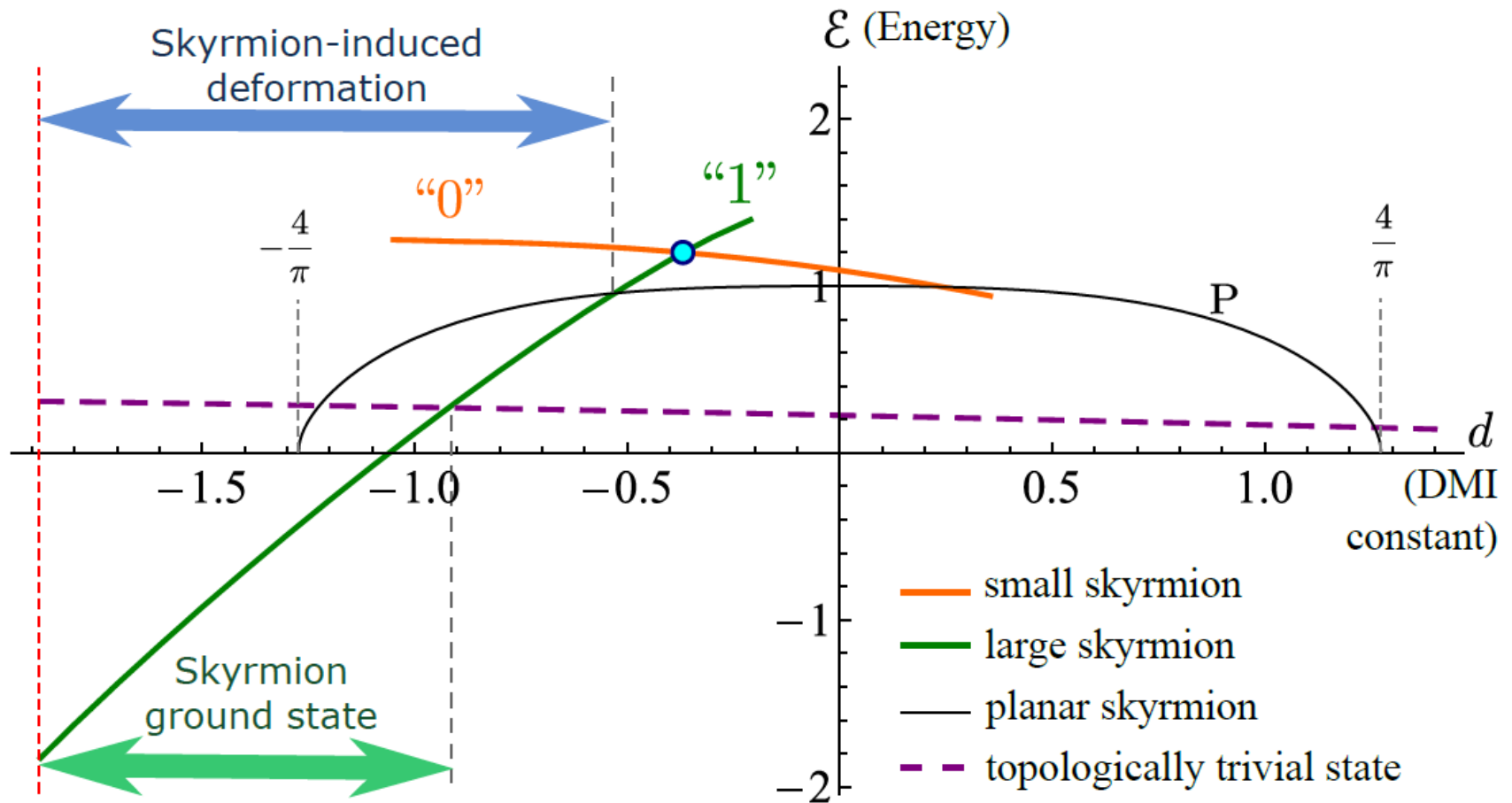$$\mathcal{A} = 2\ell, \; r_0 = \ell$$

$$L = 1 \text{ nm}$$

$$\alpha = 0.5$$

[V. Kravchuk, D. Sheka, A. Kakay, *et al.*, *Phys. Rev. Lett.* **120**, 067201 (2018)]
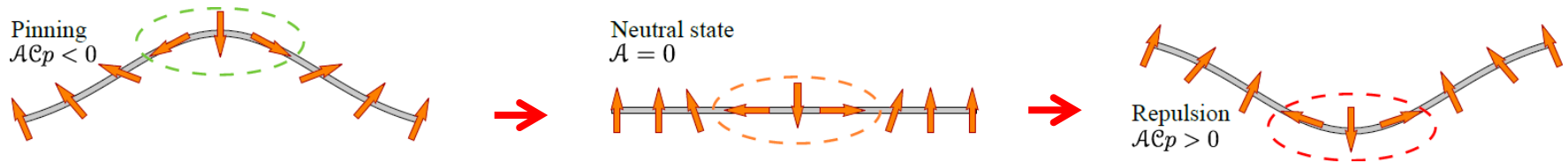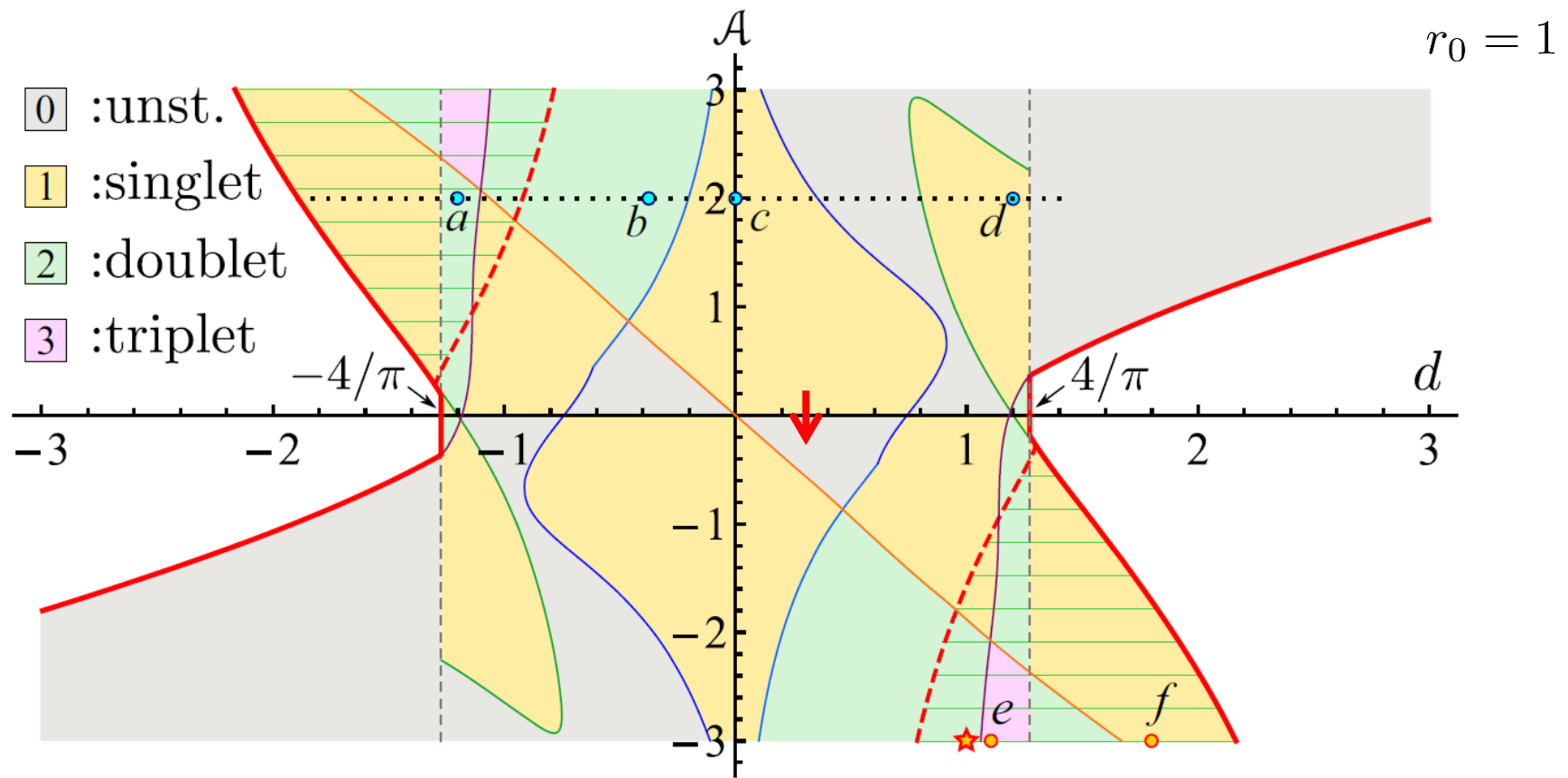
Multiplet of skyrmion states on a Gaußian bumb

$$z = \mathcal{A}e^{-r^2/(2r_0^2)}$$

$\mathcal{A} = 2,\ r_0 = 1$

[V. Kravchuk, D. Sheka, A. Kakay, *et al., Phys. Rev. Lett.* **120**, 067201 (2018)]

Multiplet of skyrmion states on a Gaußian bumb $z = \mathcal{A}e^{-r^2/(2r_0^2)}$

$r_0 = 1$

[V. Kravchuk, D. Sheka, A. Kakay, *et al.*, *Phys. Rev. Lett.* **120**, 067201 (2018)]

Conclusions and prospects

Curvature is a source of new physical phenomena in curvilinear low-dimensional magnets

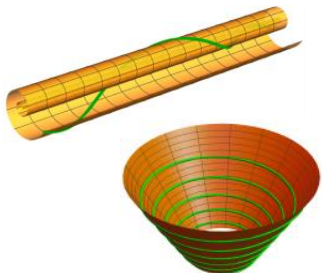**Curvature enriches physics of topological magnetic solitons:**

1. DMI-free skyrmions.
2. Skyrmion multiplets --> reconfigurable skyrmion lattices.
3. Skyrmion as a ground state.
4. Control of the vortex chirality.

Prospects

1. Curvature induced skyrmion drift
Driving along gradient of the mean curvature is expected.

2.1. Skyrmion induced film deformation
It is expected that the deformation results in skyrmion inertia mass and modifies the interaction between skyrmions.

2.2. Deformation of magnetic film controlled by the magnetization state.
DMI-induced spontaneous stripe twisting.
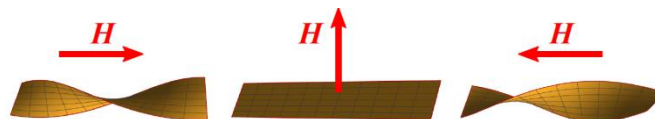
3.1 Curvilinear antiferromagnets
The curvature induced DMI and anisotropy similarly to the case of ferromagnets.

3.2. Curvilinear systems with competing exchange interactions.
New curvature induced effective interactions due to the high order terms, e.g.$(\nabla^2 m)^2$, in Hamiltonian.