Conformal phase transition in topological insulators and superconductors

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Conformal phase transition

Vladimir Miransky
The University of Western Ontario

- V. A. Miransky and Koichi Yamawaki, "Conformal phase transition in gauge theories," Phys. Rev. D 55, 5051–5066 (1997).
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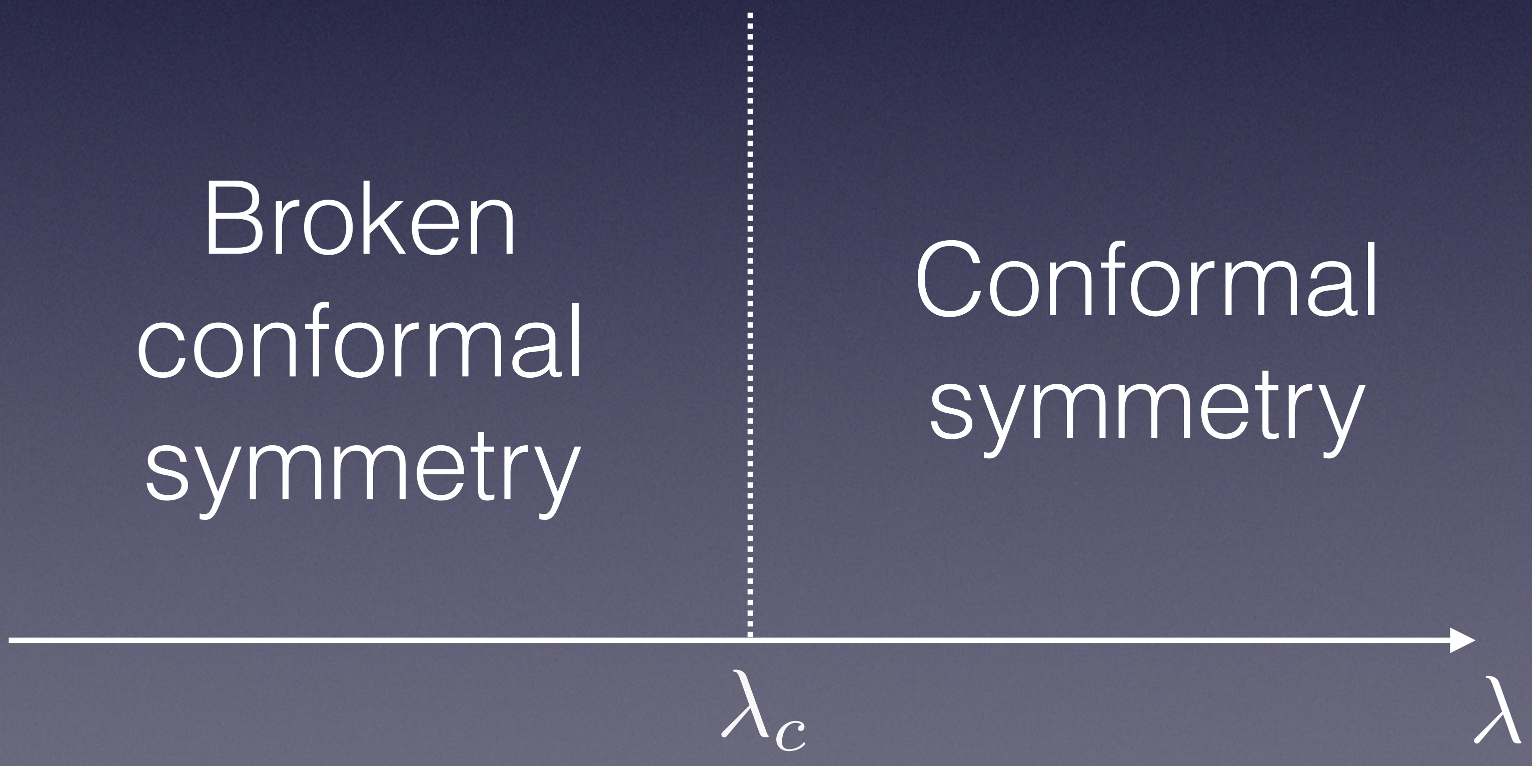
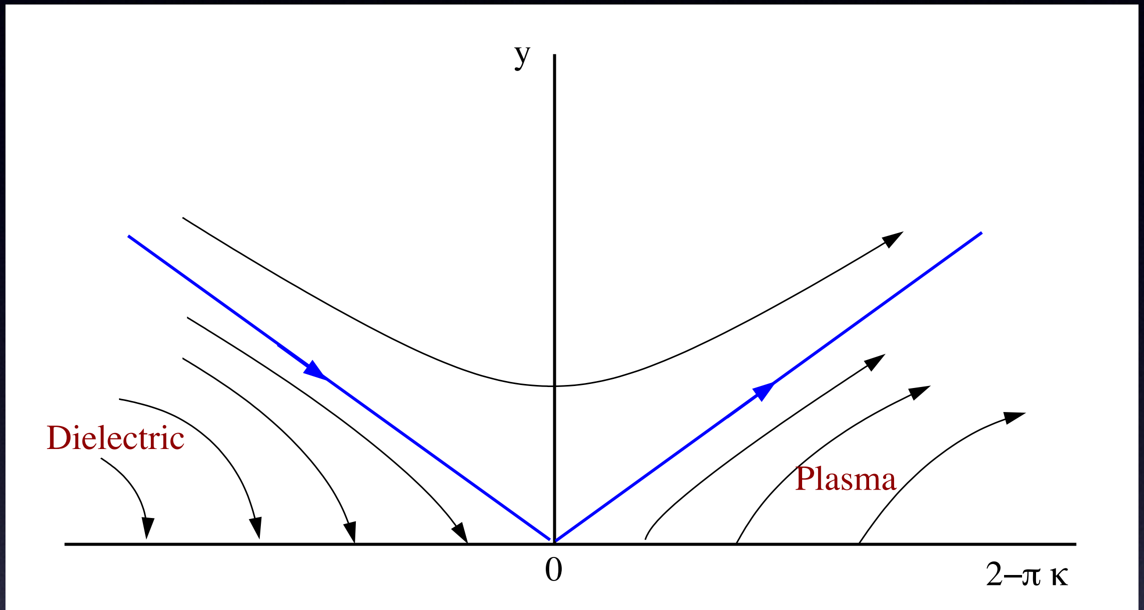


Conformal phase transition

Well known example: BKT transition

$$\xi^{-1} \sim \exp\left(-\frac{\text{const}}{\sqrt{T_c - T}}\right)$$



- Essential singularity at $T = T_c$
- No power law behavior in the correlation length

Compare typical 2nd order phase transition:

$\xi^{-1} \sim |T_c - T|^{\nu}$

Conformal phase transition

Example in 2+1 dimensions:
graphene with Coulomb interaction

E. V. Gorbar, V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, "Magnetic field driven metal-insulator phase transition in planar systems," Phys. Rev. B 66, 045108 (2002)

Excitonic gap generation by spontaneous chiral symmetry breaking:

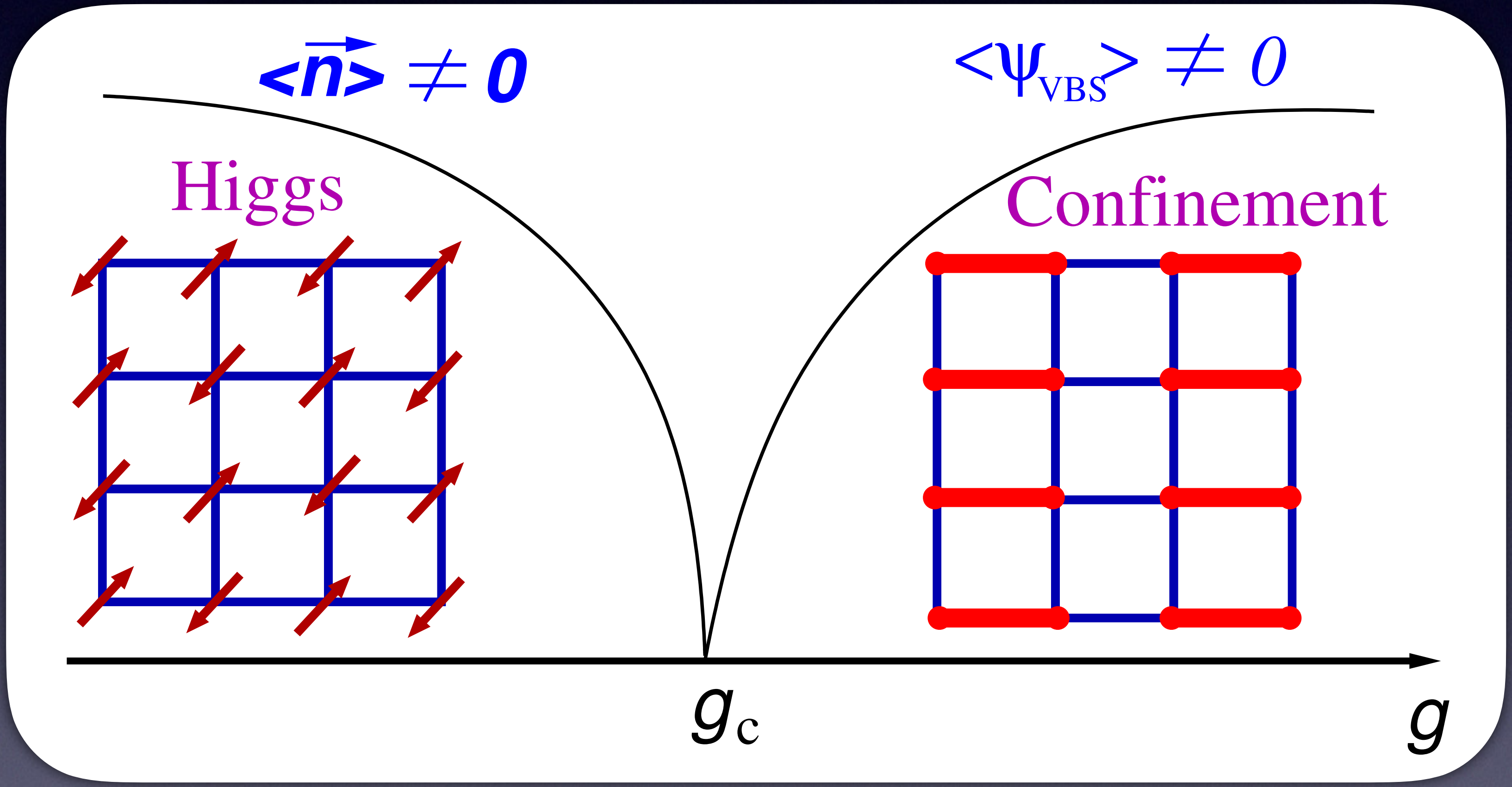
$$m \sim \exp\left(-\frac{\text{const}}{\sqrt{\alpha_c - \alpha}}\right)$$

Graphene "fine-structure constant":

$$\alpha = \frac{e^2}{\epsilon \hbar v_F}$$

Conformal phase transition

Also in d=2+1: Deconfined quantum critical points in SU(N) antiferromagnets



- Conventional 2nd order phase transition at large N
- Conformal phase transition at low N (e.g., $N = 2$)

[Nogueira and Sudbø, EPL **104**, 56004 (2013)]

Topological conformal phase transition

$$\mathcal{L} = \frac{\kappa}{2}\epsilon_{\mu\nu\lambda}A^{\mu}\partial^{\nu}A^{\lambda} - j_{\mu}A^{\mu} + \dots$$

We mainly consider two cases:

1. $j_{\mu} = \bar{\psi}\gamma_{\mu}\psi$ (Mott TI)
2. $j_{\mu} = i(\varphi^{*}\partial_{\mu}\varphi - \varphi\partial_{\mu}\varphi^{*}) + 2|\varphi|^{2}A_{\mu}$ (Top. SC)

Topological conformal phase transition

Current correlation function in conformal invariant theory in 2+1 dimensions (Euclidean):

$$\begin{aligned} C_{\mu\nu}(p) &= \langle j_\mu(p) j_\nu(-p) \rangle \\ &= \frac{1}{2\pi}\left[v|p| \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + w \epsilon_{\mu\nu\lambda} p_\lambda \right] \end{aligned}$$

v and w are universal numbers

Topological conformal phase transition

Vacuum polarization

$$C_{\mu\nu}(p) = \Pi(p)(p^2\delta_{\mu\nu} - p_\mu p_\nu) + \tfrac{\kappa}{2\pi}\epsilon_{\mu\nu\lambda}p_\lambda$$

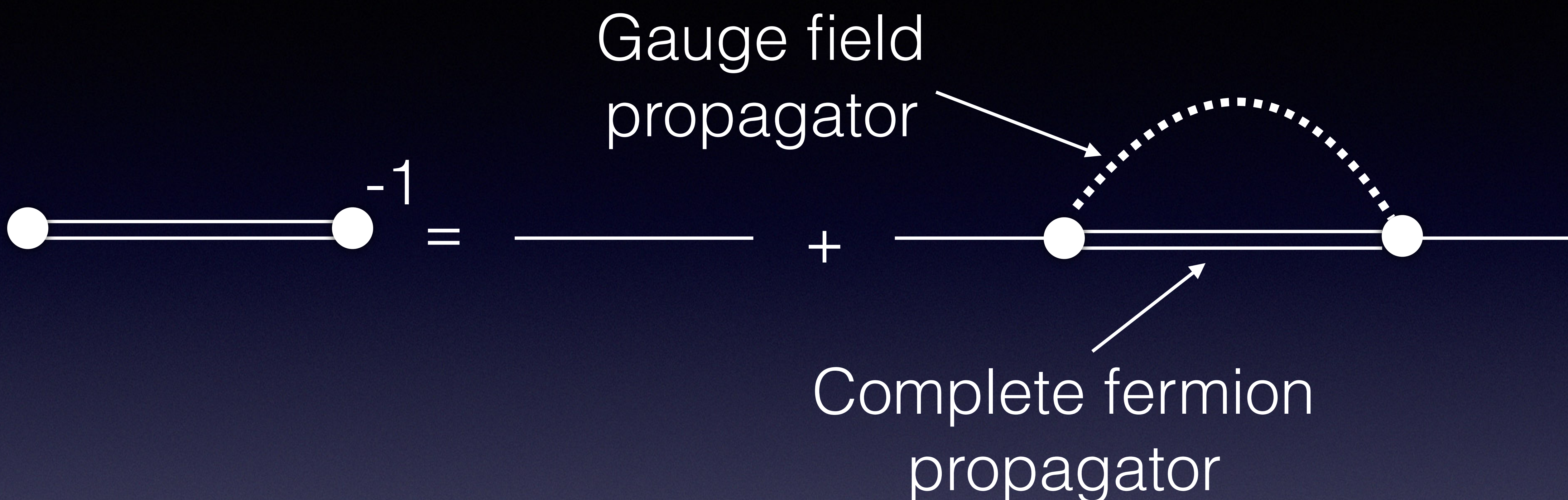
Large N_f limit: $\Pi(p) = \frac{A}{2\pi|p|}$ $\qquad A = \frac{\pi N_f}{2}$

($2N_f$ two-component fermions)

Gauge field propagator:

$$D_{\mu\nu}(p) = \frac{2\pi}{A^2+\kappa^2}\left\{\frac{A}{|p|}\left[\delta_{\mu\nu} + \frac{(\alpha-1)(A^2+\kappa^2)+\kappa^2}{A^2}\frac{p_\mu p_\nu}{p^2}\right] - \kappa\frac{\epsilon_{\mu\nu\lambda}p_\lambda}{p^2}\right\}$$

Topological conformal phase transition



Solution of the gap equation:

$$\boxed{M_{\pm} \sim \pm \exp\left[-\frac{2\pi}{\sqrt{8A/[\pi(A^2+\kappa^2)]-1}}\right]}$$

Topological conformal phase transition

Physical interpretation:

$$\langle(\psi^\dagger_{a\uparrow}\psi_{a\uparrow} - \psi^\dagger_{a\downarrow}\psi_{a\downarrow})\rangle \sim M_+$$

$$\langle(\chi^\dagger_{a\uparrow}\chi_{a\uparrow} - \chi^\dagger_{a\downarrow}\chi_{a\downarrow})\rangle \sim M_-$$

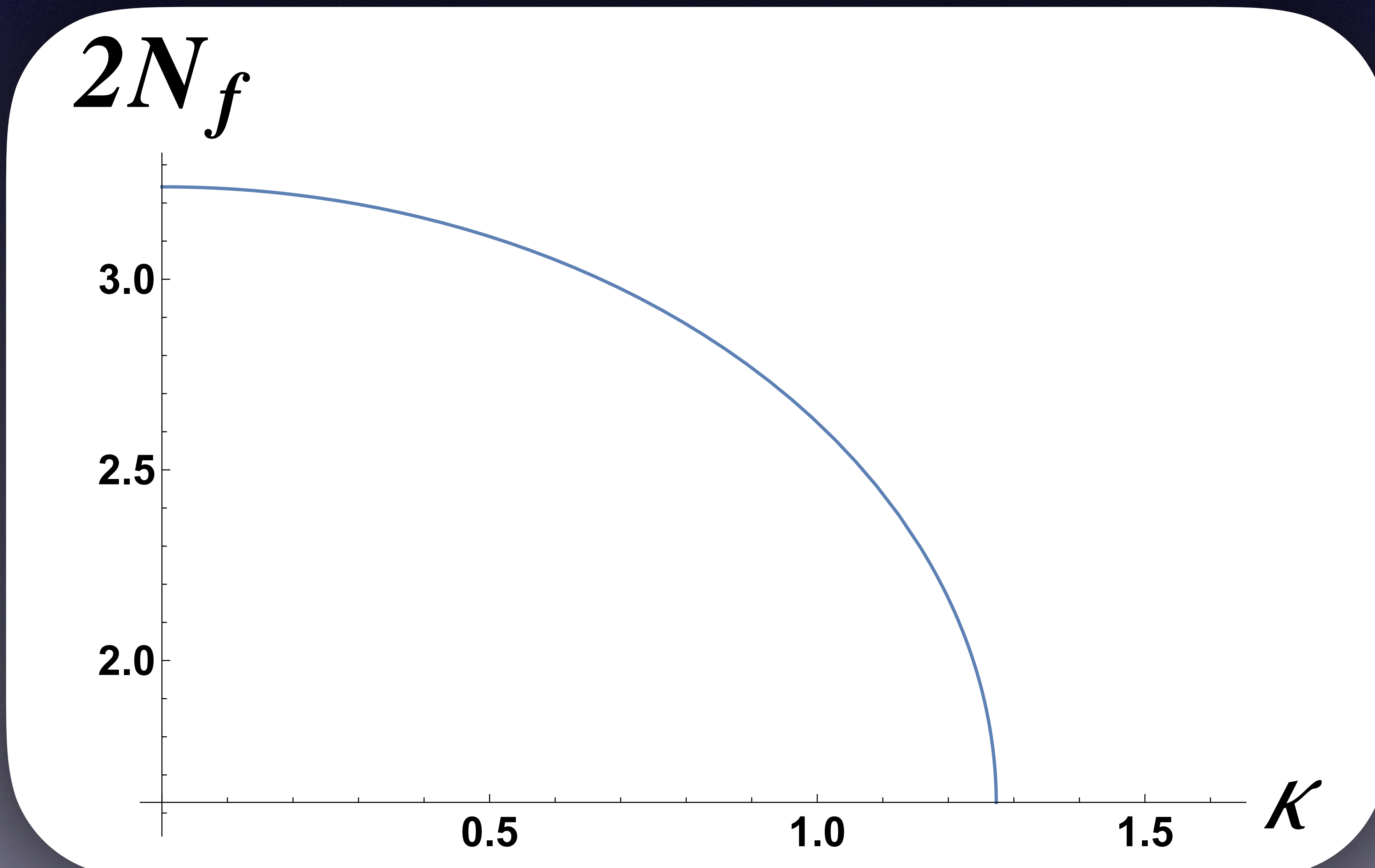
$$a = 1, \ldots, N_f$$

Chiral symmetry breaking $\Longrightarrow$ Spin-density wave state

Topological conformal phase transition

$$M_\pm \neq 0 \Longrightarrow A > \frac{4}{\pi}\left[1 + \sqrt{1 - \left(\frac{\pi\kappa}{4}\right)^2}\right]$$

$$\Longrightarrow N_f = 1 \text{ and } |\kappa| < \kappa_c = 4/\pi \approx 1.27$$



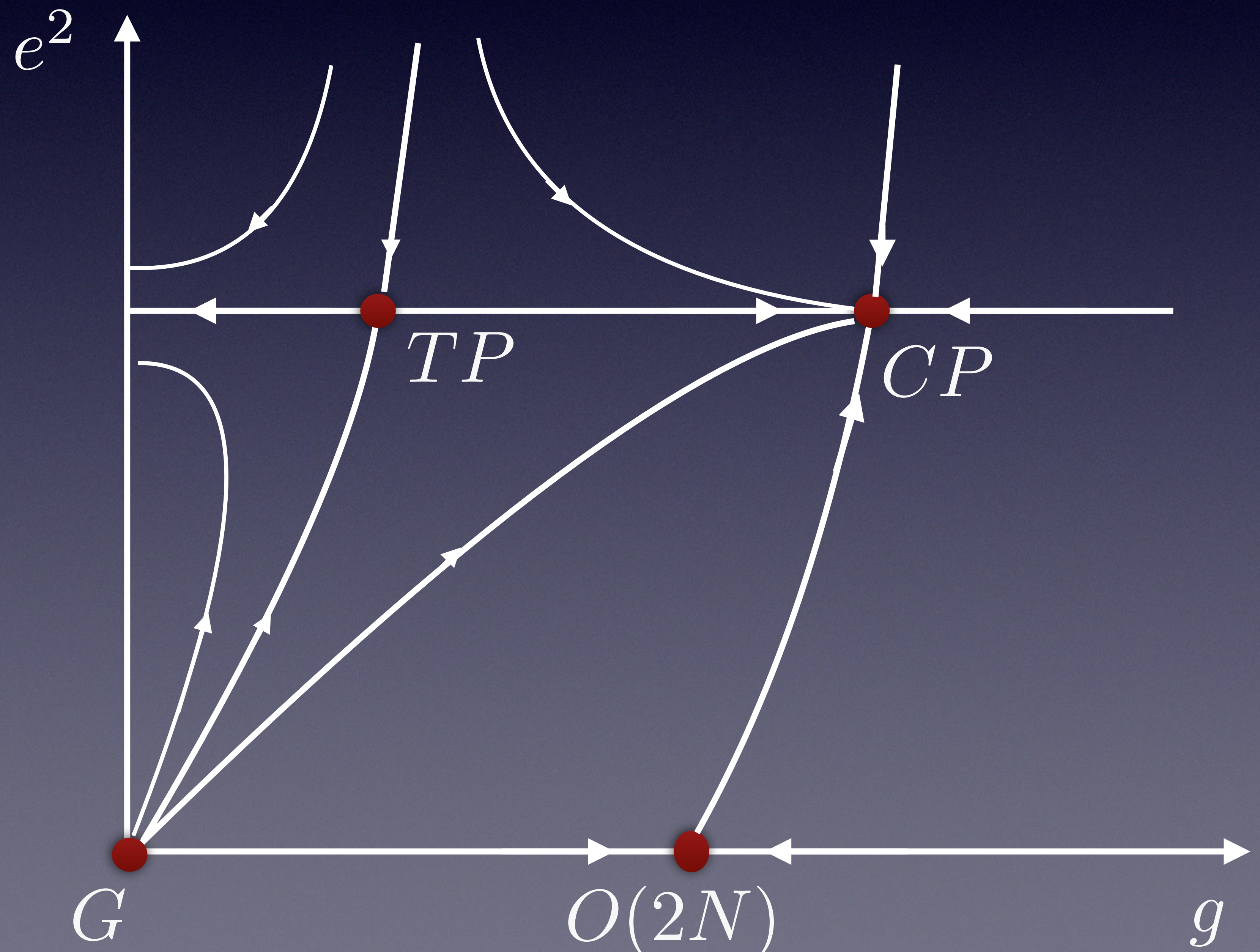
Topological conformal phase transition

Superconductor Chern-Simons-Higgs electrodynamics

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{4e^2}F_{\mu\nu}^2 + |(\partial_\mu - iA_\mu)\varphi|^2 + m^2|\varphi|^2 + \frac{g}{2}|\varphi|^4 \\
&+ i\frac{\kappa}{2}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda
\end{aligned}
$$

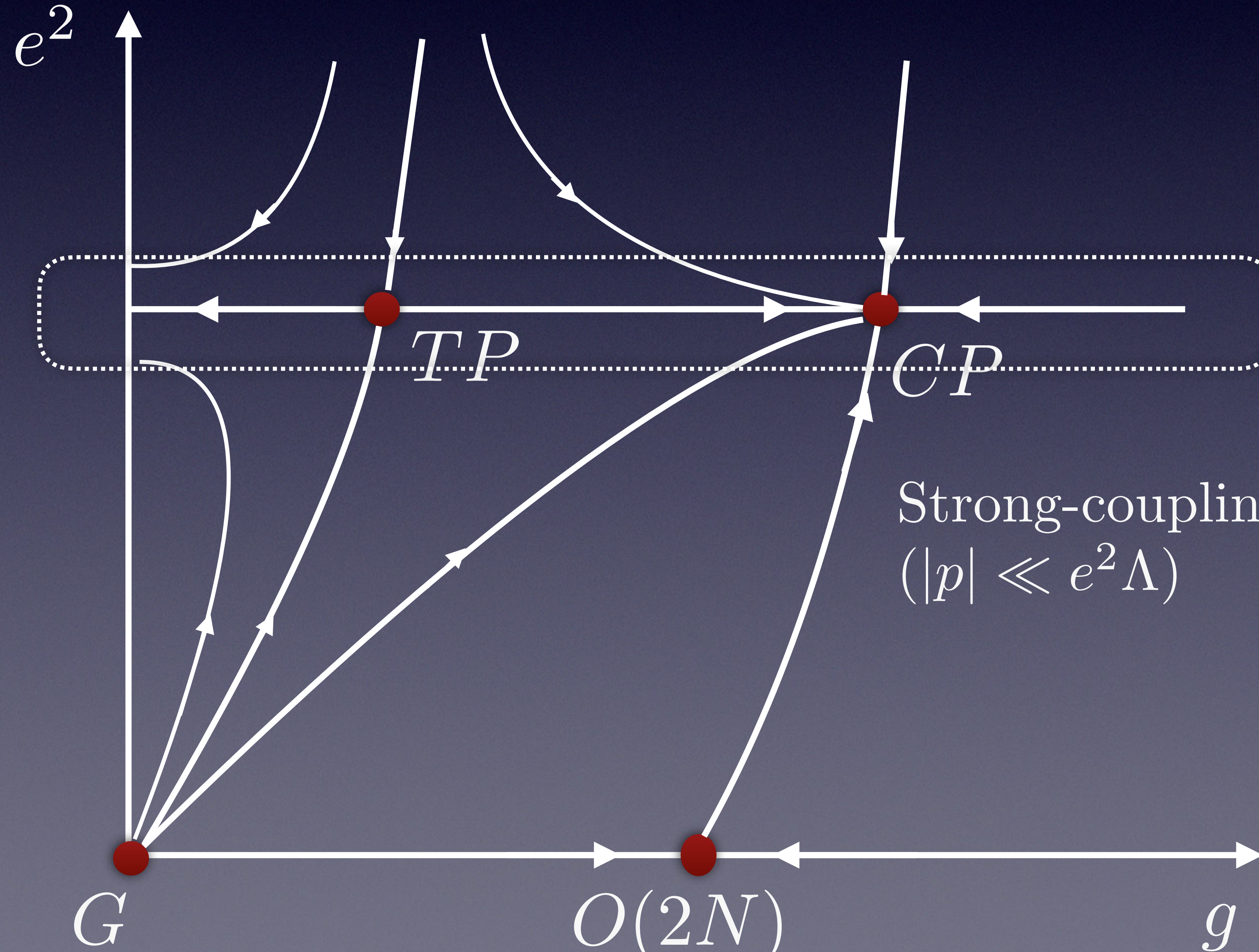
Topological conformal phase transition

Renormalization group flow



Topological conformal phase transition

Renormalization group flow



Topological conformal phase transition

RG equation

$$\beta(g) = \mu \frac{dg}{d\mu} = \left(\frac{N+4}{8}\right) \left[-\Omega + (g - g_*)^2\right]$$

$$g_* = \frac{4}{N+4} \left(\frac{8\kappa}{3\pi^2 N} - 1 - \frac{8}{N}\right)$$

$$\Omega = \left(\frac{8\kappa}{3\pi^2 N} - 1 - \frac{8}{N}\right)^2 - \frac{48}{N} \left(1 + \frac{4}{N}\right)$$

- Quantum criticality: $\Omega > 0$
- Conformality lost: $\Omega < 0$
  $\Longrightarrow g_\pm = g_* \pm i\sqrt{|\Omega|}$

Topological conformal phase transition

Conformality lost phase:

$$\frac{3\pi^2}{8}\left[N+8-4\sqrt{3(N+4)}\right] < \kappa < \frac{3\pi^2}{8}\left[N+8+4\sqrt{3(N+4)}\right]$$

$$\xi^{-1} \sim \Lambda_{IR}$$

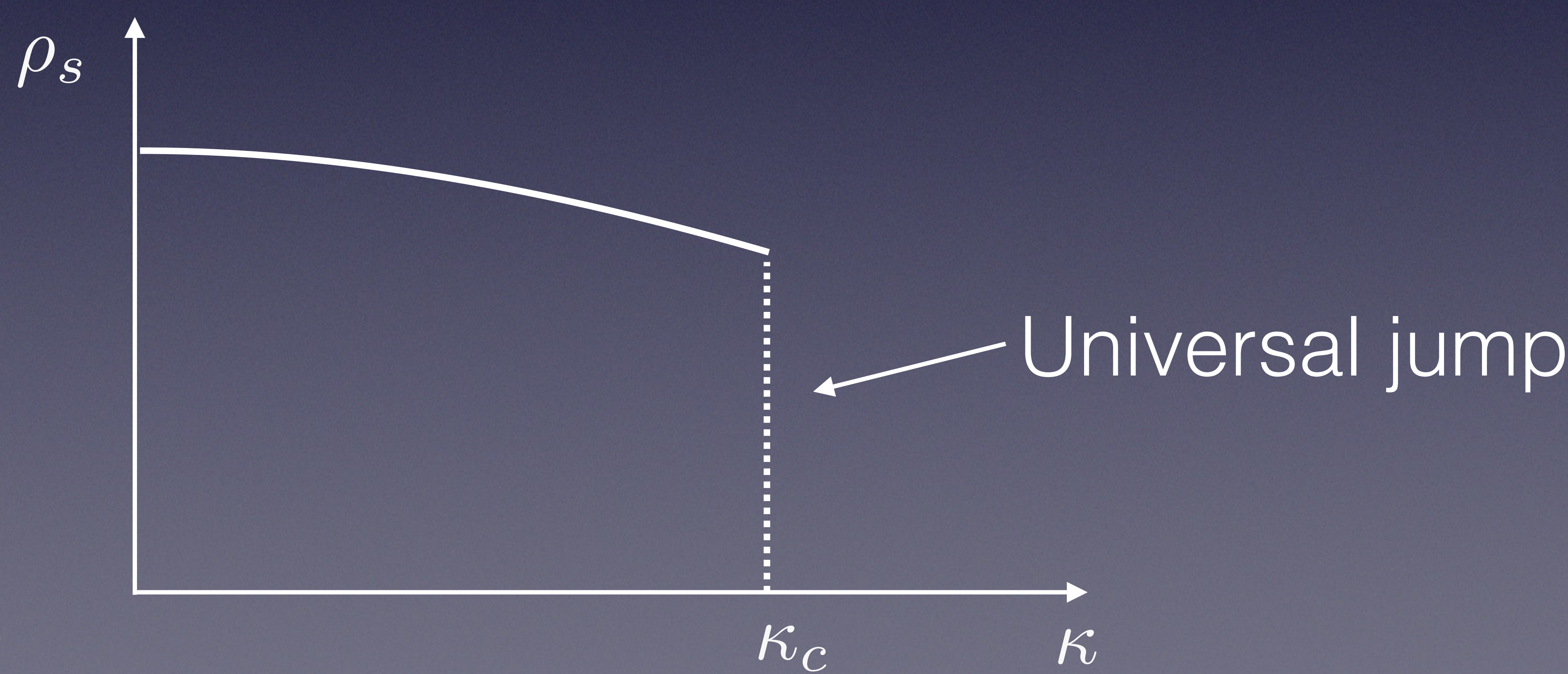
$$\boxed{\frac{\Lambda_{IR}}{\Lambda_{UV}} \approx \exp\left[-\frac{8\pi}{(N+4)\sqrt{|\Omega|}}\right]}$$

Topological conformal phase transition

Physical implications

RG equation for superfluid stiffness:

$$\mu \frac{d\rho_s}{d\mu} = \left[1 - \frac{\beta(g)}{g}\right]\rho_s$$



Conclusions

- Conformal phase transitions leads to BKT-like scaling in higher dimensions
- Unconventional quantum critical behavior
- Mott TI: CSB induced by Chern-Simons term
- Top. SC: fluctuation induced Higgs mechanism with BKT scaling