Dynamics of magnetic domain walls in curvilinear wires

Kostiantyn Yershov
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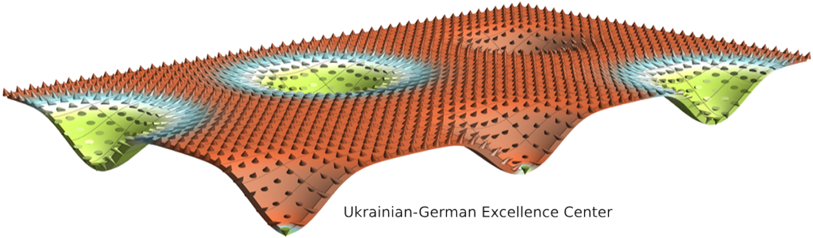

UKRATOP Workshop

December 04, 2018

Outline

1. Phenomenological model of the ferromagnet and general approach for curvilinear wires.
2. Curvature effects in domain wall dynamics.
3. Torsion effects in domain wall dynamics.
4. Conclusions.

Phenomenological model of the ferromagnet

Magnetization equation of motion

$$\frac{\partial \vec{m}}{\partial \bar{t}} = \vec{m} \times \frac{\delta \mathcal{E}}{\delta \vec{m}} + \alpha\, \vec{m} \times \frac{\partial \vec{m}}{\partial \bar{t}}$$

$\vec{m} = \vec{M}/M_s$ – unit magnetization vector, M_s – saturation magnetization,
α – Gilbert damping, $\mathcal{E} = E/K$ – dimensionless energy,
$\bar{t} = t\omega_0$ – dimensionless time, $\Omega_0 = \gamma_0 K/M_s$.

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Energy of the ferromagnet

$$E = \underbrace{-A \int_V \left(\vec{m} \cdot \nabla^2 \vec{m}\right) d\vec{r}}_{\text{Exchange energy } E_{\text{ex}}} \quad - \quad \underbrace{K \int_V \left(\vec{m} \cdot \vec{e}_{\text{A}}\right)^2 d\vec{r}}_{\text{Anisotropy } E_{\text{a}}} \quad \underbrace{- \frac{M_s}{2} \int_V \left(\vec{m} \cdot \vec{h}_{\text{ms}}\right) d\vec{r}}_{\text{Magnetostatic energy } E_{\text{ms}}}$$

A – exchange constant, K – anisotropy constant, $\vec{h}_{\text{ms}}$ – stray field

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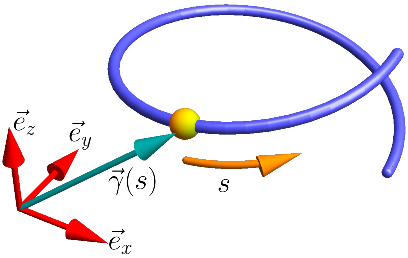
$\ell = \sqrt{A/K} \sim 10$ nm – magnetic length, $\qquad E_{\textsf{ms}} \Rightarrow E_{\textsf{a}}^{\textsf{eff}} = -K^{\textsf{eff}} \int_V \left(\vec{m} \cdot \vec{e}_{\rm A}\right)^2 d\vec{r}$

$K_{1\rm D}^{\textsf{eff}} = \pi M_s^2$ – effective easy axial anisotropy constant
[Slastikov, Sonnenberg, Journal of Applied Mathematics **77**, 220, (2012)]

Exchange energy in curvilinear wire

Frennet-Serret formulas

$$\vec{\gamma} = \vec{\gamma}\,(s) \text{ – curvilinear wire}$$



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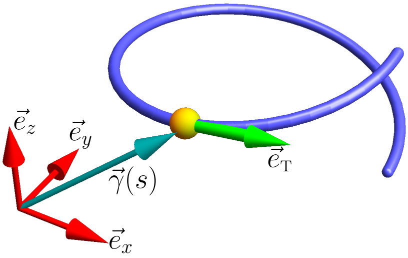
$\vec{e}_{\mathrm{T}} = \vec{\gamma}'\,(s)$



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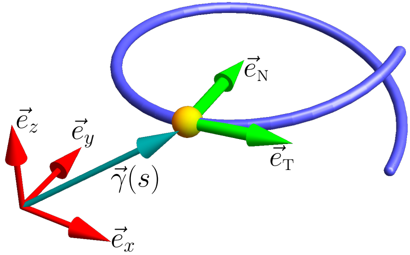

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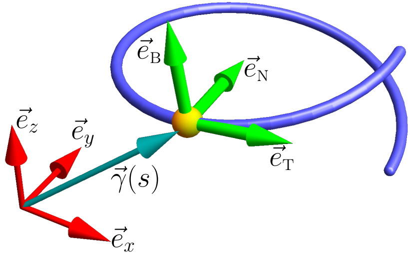

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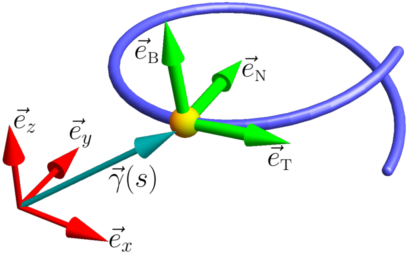
$$\{\mu, \nu\} = \{\mathrm{T,N,B}\}$$

κ – curvature, τ – torsion



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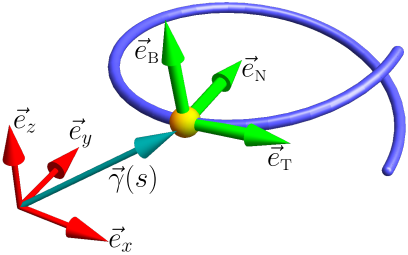

Exchange energy in curvilinear basis

$$\mathscr{E}_{\text{ex}} = (\vec{e}_{\mu} m_{\mu})' \, (\vec{e}_{\nu} m_{\nu})'$$

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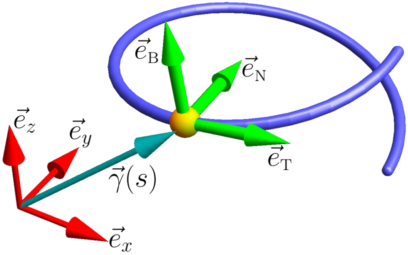

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$$\mathscr{E}_{\text{ex}} = (\vec{e}_\mu m_\mu)'(\vec{e}_\nu m_\nu)' = \mathscr{E}_{\text{ex}}^0 + \mathscr{E}_{\text{ex}}^{\text{DMI}} + \mathscr{E}_{\text{ex}}^{\text{A}}$$

[Sheka, Kravchuk, Gaididei, J. Phys. A 48, 125202, (2015)]

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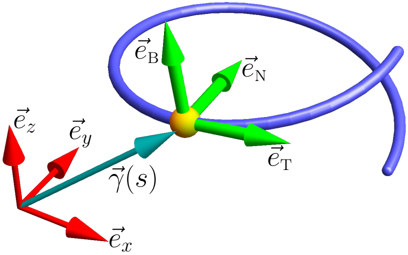

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$$\underbrace{\mathscr{E}_{\text{ex}}^0 = |\vec{m}'|^2,}_{\text{Isotropic eschange}} \quad \underbrace{\mathscr{E}_{\text{ex}}^{\mathrm{DMI}} = \mathscr{F}_{\mu\nu}\left(m_\mu m_\nu' - m_\mu' m_\nu\right),}_{\text{Effective Dzyaloshinskii-Moria interaction}} \quad \underbrace{\mathscr{E}_{\text{ex}}^{\mathrm{A}} = \mathscr{K}_{\mu\nu} m_\mu m_\nu,}_{\text{Effective anisotropy}} \quad \mathscr{K}_{\mu\nu} = \mathscr{F}_{\mu\zeta}\mathscr{F}_{\nu\zeta}$$

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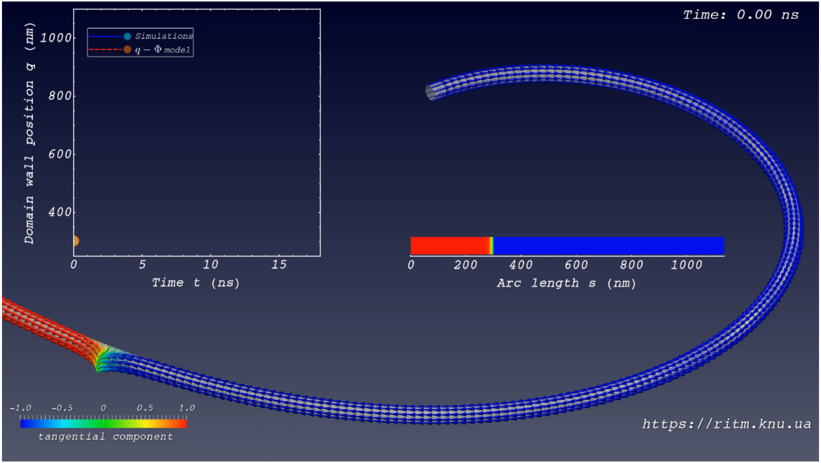
Curvature induced motion of the domain wall (Cornu spiral as an example)



[Yershov, Kravchuk, Sheka, Pylypovskyi, Makarov, Gaididei, PRB **98**, 060409(R) (2018)]

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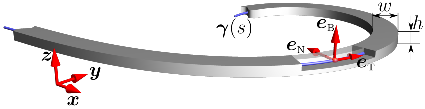
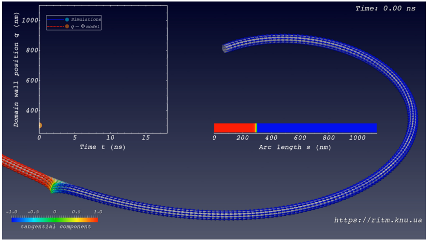


Energy of the system

$$\mathcal{E} = \ell^2 \mathcal{E}_{\textbf{ex}} - (\vec{m} \cdot \vec{e}_{\mathrm{T}})^2 + \epsilon \, (\vec{m} \cdot \vec{e}_{\mathrm{B}})^2$$

$\epsilon > 0$ – anisotropy ratio.



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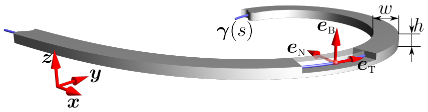

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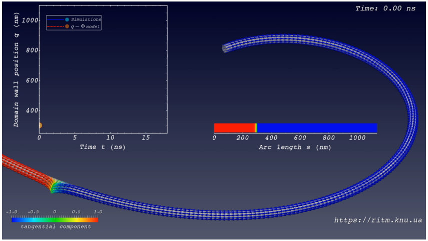

Collective variable approach (Slonzewski $q - \Phi$ model) $\Rightarrow$ domain wall velocity

$$V = -p\mathcal{C}\frac{\Delta_0 \omega_0}{\alpha}\chi\ell^2$$

$p = \pm 1$ – DW topological charge, $\mathcal{C} = \pm 1$ DW chirality, Δ_0 – DW width,
$\chi = \kappa' =$const – gradient of the curvature.

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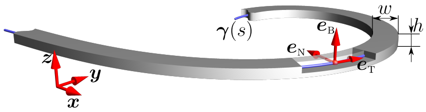

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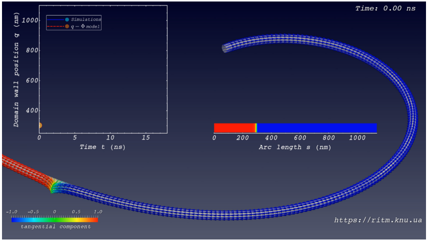

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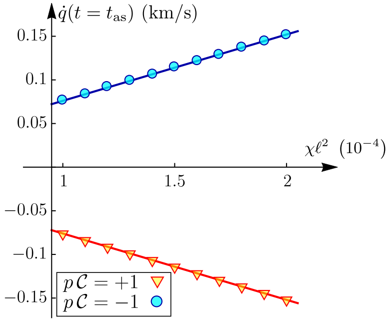

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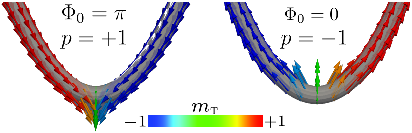
Domain wall pinning at a local wire bend $[\kappa(\pm\infty) = 0]$



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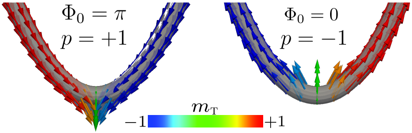

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Collective variable approach (Slonzewski $q - \Phi$ model) $\Rightarrow$ harmonic oscillations

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$$\eta \approx \alpha\omega_0 \frac{\pi}{2} \frac{\ell^2 \kappa(q_0)}{\Delta} \text{ – Effective damping}$$

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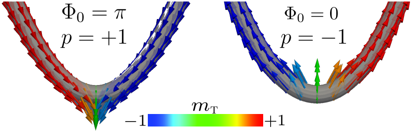

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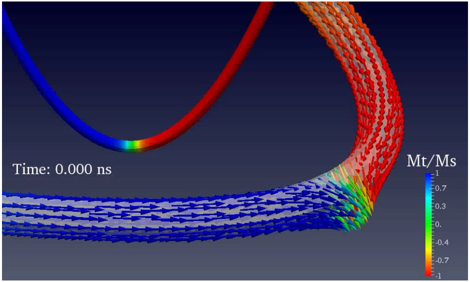

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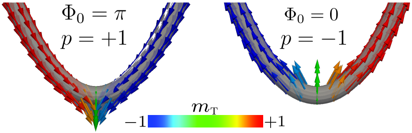

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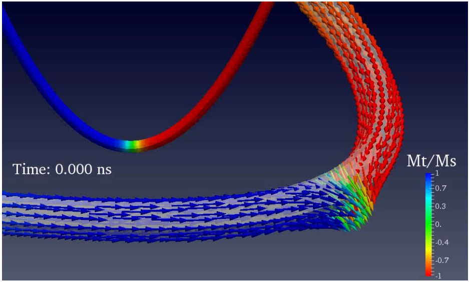

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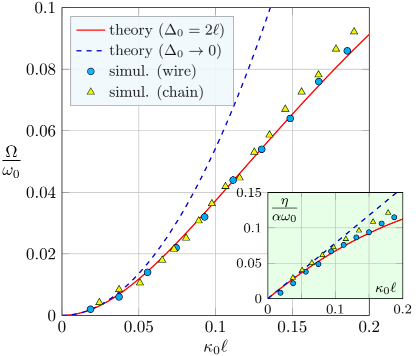

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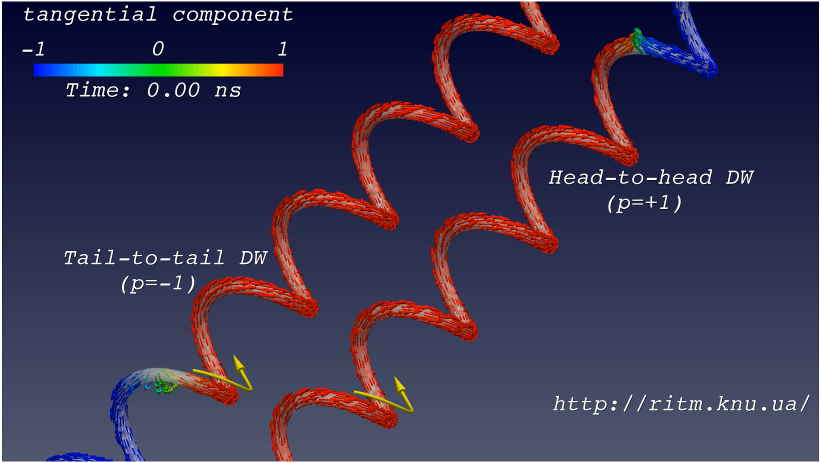
Torsion effects in current driven domain wall motion



[Yershov, Kravchuk, Sheka, Gaididei, PRB 93, 094418 (2016)]

Torsion effects in current driven domain wall motion

Collective variable approach (Slonzewski $q-\Phi$ model) $\Rightarrow$ equations of motion

$$\dot{q} - \alpha p \Delta_0 \dot{\Phi} = u - \pi\kappa \sin\Phi,$$

$$\alpha p \dot{q} + \Delta_0 \dot{\Phi} = up\left(\beta - \beta^\star\right), \quad \beta^\star = p\tau\Delta_0$$

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- Curvature induced Walker limit

  $$u_\mathrm{W} \approx v_0 \alpha\pi\kappa\ell / \left(\alpha - \beta + \beta^\star\right), \quad v_0 = \omega_0 \ell$$

[Yershov, Kravchuk, Sheka, Gaididei, PRB 93, 094418 (2016)]

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- Torsion induced effective nonadiabaticy shift

$$\beta \to \beta - \beta^\star, \quad \beta^\star = p\Delta_0\tau$$

Types of DW motion

- Traveling wave solution ($u < u_{\mathrm{w}}$)

$$V = u\left(\beta - \beta^\star\right)/\alpha$$

[Yershov, Kravchuk, Sheka, Gaididei, PRB 93, 094418 (2016)]

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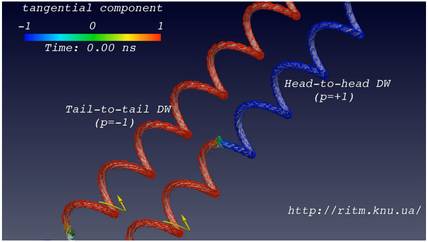

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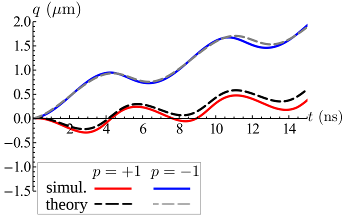
Types of DW motion

- Traveling wave solution ($u < u_{\mathrm{W}}$)

$$V = u\left(\beta - \beta^\star\right)/\alpha$$

- Precession motion ($u > u_{\mathrm{W}}$)

$$\Omega = p\left(\beta - \beta^\star - \alpha\right)\sqrt{u^2 - u_{\mathrm{W}}^2}/\Delta_0$$



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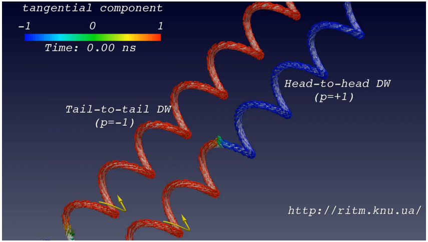
- Torsion induced effective nonadiabaticy shift

$$\beta \to \beta - \beta^\star, \quad \beta^\star = p\Delta_0\tau$$



[Yershov, Kravchuk, Sheka, Gaididei, PRB 93, 094418 (2016)]

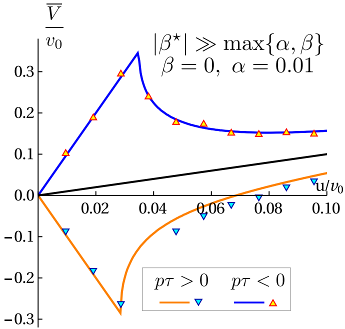
Types of DW motion

- Traveling wave solution ($u < u_{\mathrm{W}}$)

$$V = u\left(\beta - \beta^\star\right)/\alpha$$

- Precession motion ($u > u_{\mathrm{W}}$)

$$\Omega = p\left(\beta - \beta^\star - \alpha\right)\sqrt{u^2 - u_{\mathrm{W}}^2}/\Delta_0$$



Conclusions

- Gradient of the curvature results in a driving force for the domain wall
- Local wire bend results in a pinning potential for the domain wall
- Curvature results in the Walker limit appearance in uniaxial magnet
- Torsion effectively shifts the material parameter of nonadiabaticity and results in negative domain wall mobility

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Thank You For Your Attention!

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