

Resonance X-ray reflectivity — a tool to extract valence profiles and atomic scattering factors

Volodymyr Zabolotnyy

Dresden, December 2018

Experimental Setup

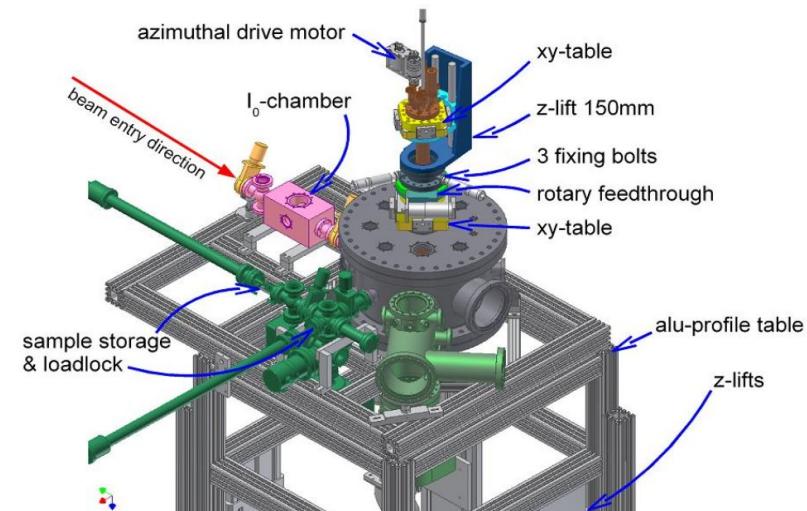
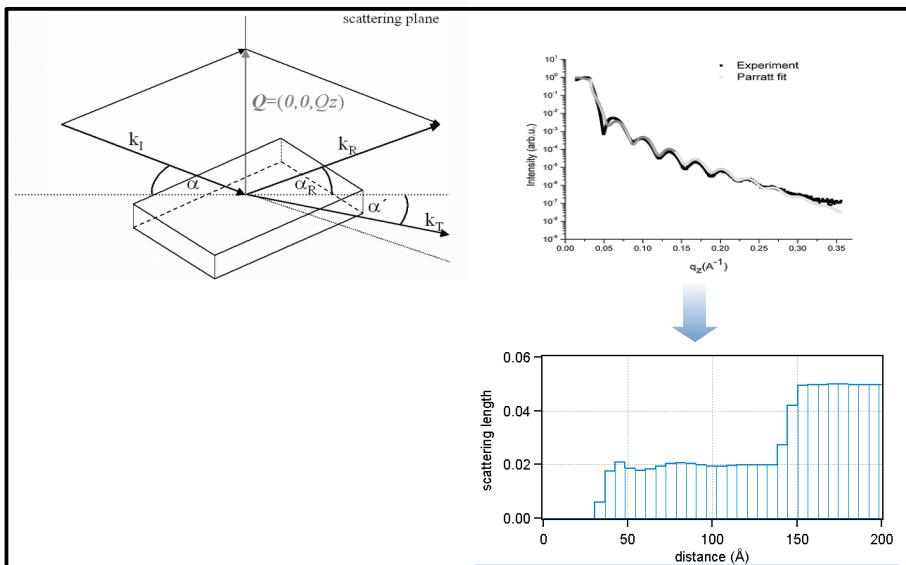


Image <http://www.is.mpg.de>

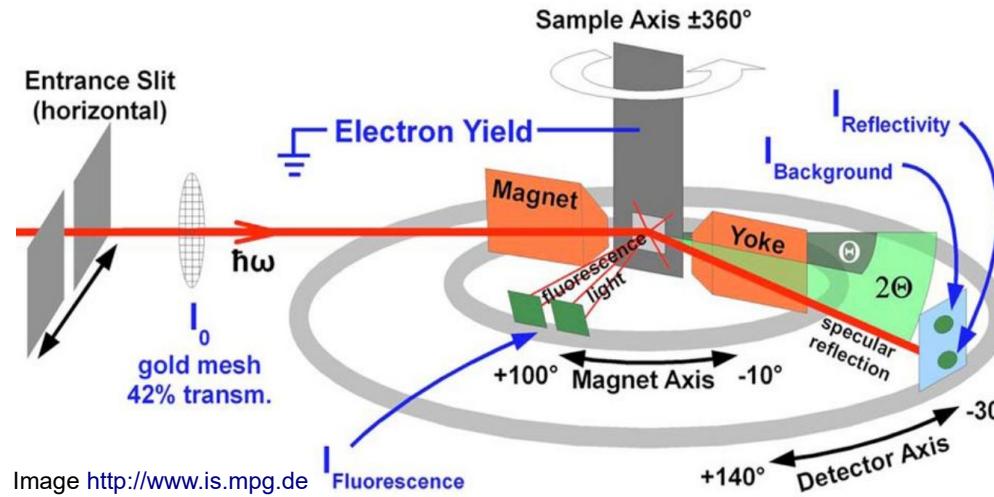
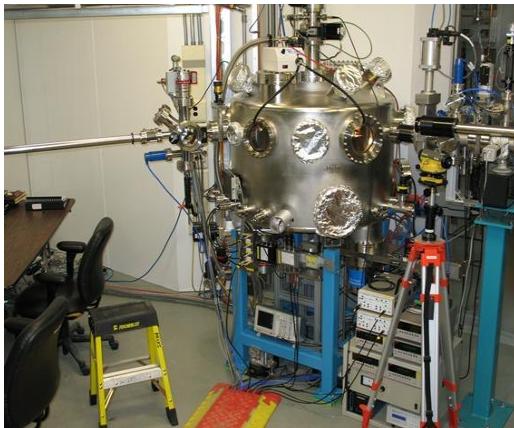
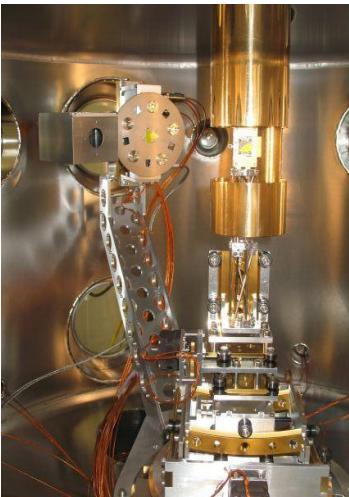


Image <http://www.is.mpg.de>

REIX beamline at CLS



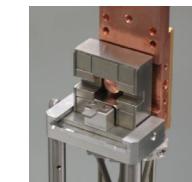
RSXS Scattering Chamber



In-Vacuum Diffractometer

RSXS Endstation

1. (Resonant) Soft X-ray Scattering (RSXS) / X-Ray reflectometry
 - 10-motion UHV diffractometer (4-circle diffractometer + x,y,z and detector motions)
 - Detectors: Channeltron, Photodiode, with multiple slits and filters, 2D Micro-Channelplate (MCP), Polarization Analyzer, Multi-channel Scaler, Electrometer.
2. X-ray Absorption Spectroscopy (XAS)
 - by total electron yield (TEY), total fluorescence yield (TFY)
3. Magnetic Circular Dichroism (MCD)
 - full polarization control of the incoming beam.



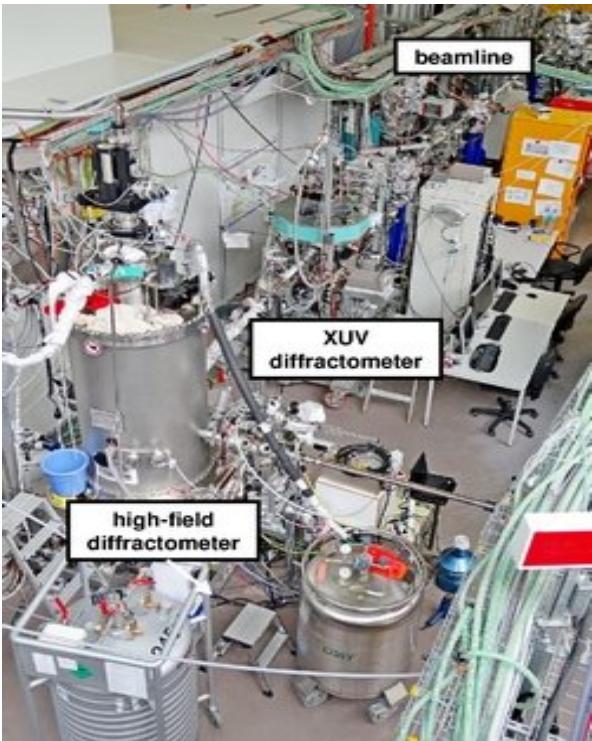
Permanent magnet

Status	RSXS Endstation: Operational, Accepting Proposals XES Endstation: Accepting Special Request
Source	Elliptically Polarizing Undulator (EPU)
Energy Range	80 – 2000 eV
Wavelength	155 – 6.2 Å
Resolution $\Delta E / E @ E$	$5 \times 10^{-5} @ 100 \text{ eV}$ $1.3 \times 10^{-4} @ 1000 \text{ eV}$
Flux (y/s/0.1%BW) @ 100 mA	$1 \times 10^{12} @ 100 \text{ eV}$ $5 \times 10^{11} @ 1000 \text{ eV}$
Spot size (HxV)	RSXS: 250 x 150 µm XES: 60 x 10 µm



He4 closed cycle cryostat

UE46-PGM1 beamline at BESSY

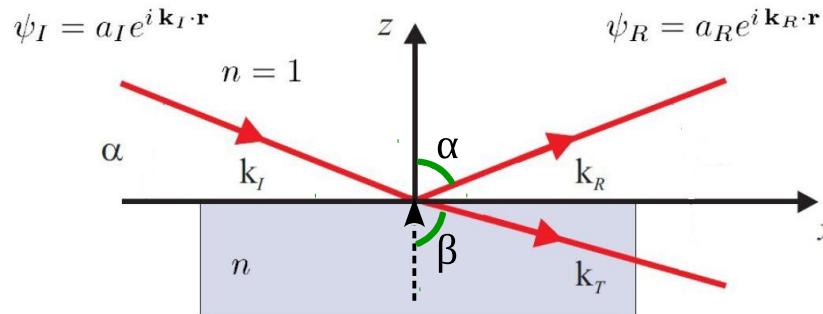


Selected Applications

- Resonant diffraction from magnetic, charge, and orbital order superstructures
- Spectroscopy of electronic ordering phenomena
- Magnetization states of single molecular magnets
- Element-specific magnetic hysteresis loops

Calculating reflectivity from a stack

A. Single interface \Rightarrow Fresnel equations



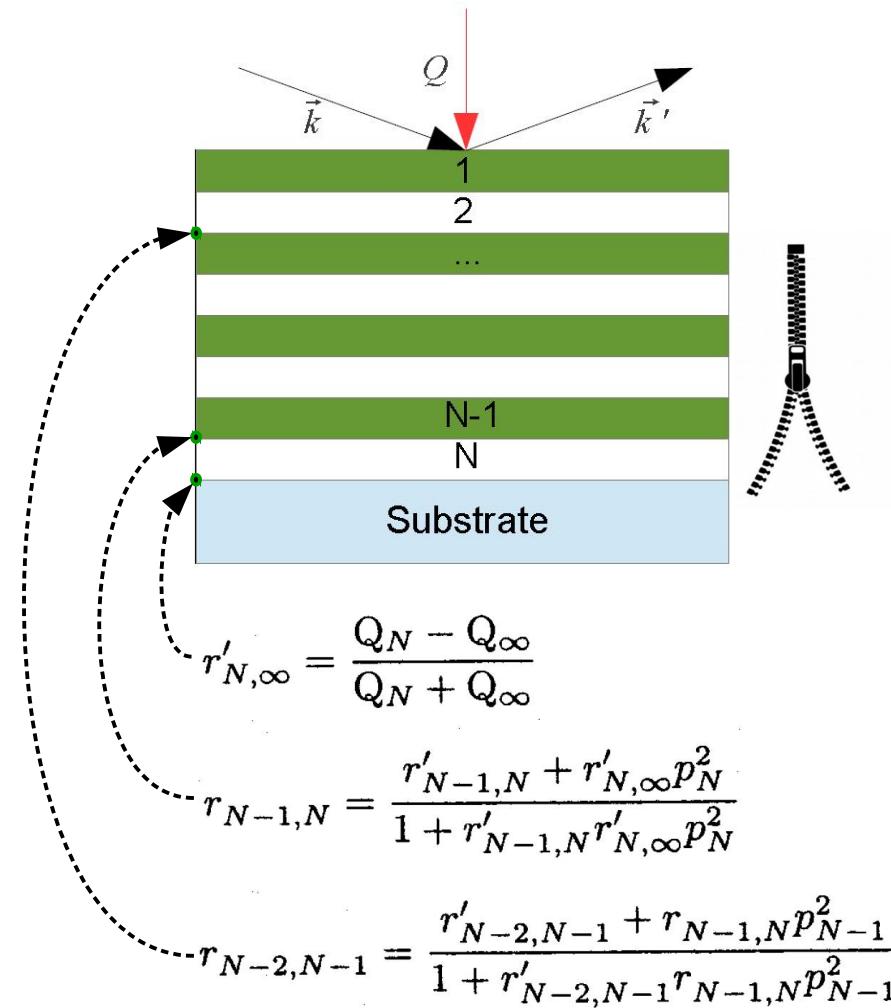
$$\left(\frac{E_{0t}}{E_{0e}}\right)_s = t_s = \frac{2n_1 \cos \alpha}{n_1 \cos \alpha + n_2 \cos \beta}$$

$$\left(\frac{E_{0r}}{E_{0e}}\right)_s = r_s = \frac{n_1 \cos \alpha - n_2 \cos \beta}{n_1 \cos \alpha + n_2 \cos \beta}$$

$$\left(\frac{E_{0t}}{E_{0e}}\right)_p = t_p = \frac{2n_1 \cos \alpha}{n_2 \cos \alpha + n_1 \cos \beta}$$

$$\left(\frac{E_{0r}}{E_{0e}}\right)_p = r_p = \frac{n_2 \cos \alpha - n_1 \cos \beta}{n_2 \cos \alpha + n_1 \cos \beta}$$

B. Thick sample \Rightarrow iterative Parrat approach



Calculating reflectivity from a stack

C. Thick sample, arbitrary polarization+anisotropic $\epsilon(z) \Rightarrow$ matrix approach

Surface Science 96 (1980) 41–53
 © North-Holland Publishing Company

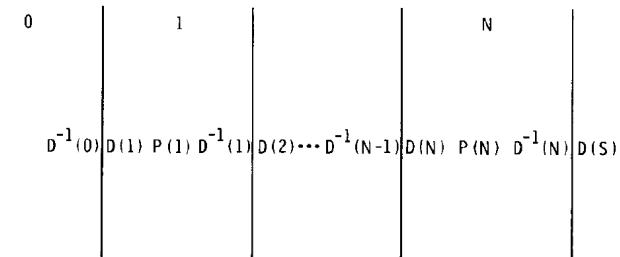
OPTICS OF ANISOTROPIC LAYERED MEDIA: A NEW 4×4 MATRIX ALGEBRA

Pochi YEH

Rockwell International Science Center, Thousand Oaks, California 91360, USA

Received 20 August 1979

S
U
R
F
A
C
E



Continuity condition:

$$D(n-1) \begin{pmatrix} A_1(n-1) \\ A_2(n-1) \\ A_3(n-1) \\ A_4(n-1) \end{pmatrix} = D(n) P(n) \begin{pmatrix} A_1(n) \\ A_2(n) \\ A_3(n) \\ A_4(n) \end{pmatrix}$$

$$\begin{pmatrix} A_1(n-1) \\ A_2(n-1) \\ A_3(n-1) \\ A_4(n-1) \end{pmatrix} = \underbrace{T_{0,1} T_{1,2} T_{2,3} \dots T_{N-1,N} T_{N,s}}_M \begin{pmatrix} A_1(s) \\ A_2(s) \\ A_3(s) \\ A_4(s) \end{pmatrix}$$

M

incident σ $\rightarrow \begin{pmatrix} A_s \\ B_s \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} C_s \\ 0 \\ C_p \\ 0 \end{pmatrix} \rightarrow$ transmitted σ

reflected σ $\leftarrow \begin{pmatrix} A_s \\ B_s \end{pmatrix}$

incident π $\rightarrow \begin{pmatrix} A_p \\ B_p \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} C_s \\ 0 \\ C_p \\ 0 \end{pmatrix} \rightarrow$ transmitted π

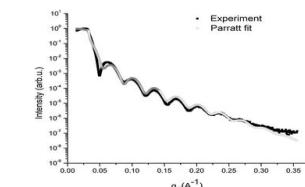
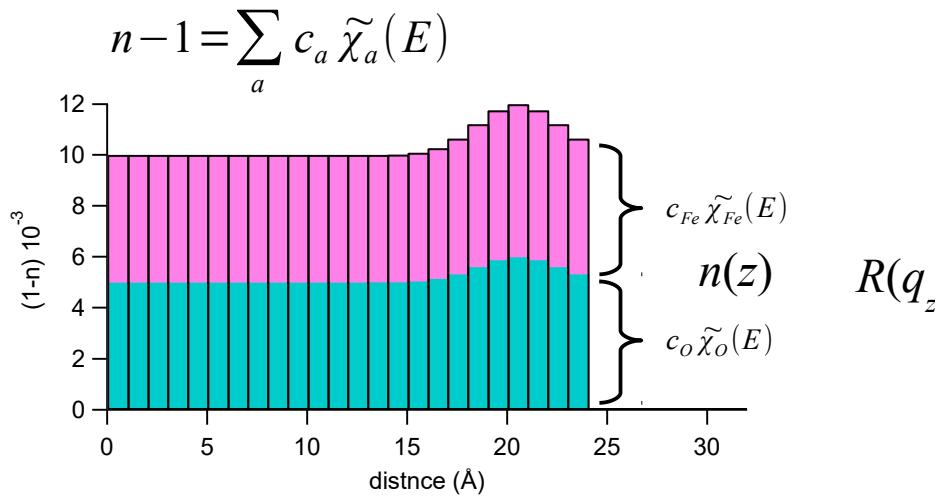
reflected π $\leftarrow \begin{pmatrix} A_p \\ B_p \end{pmatrix}$

S
U
B
S
T
R
A
T
E

Where does element sensitivity comes from?

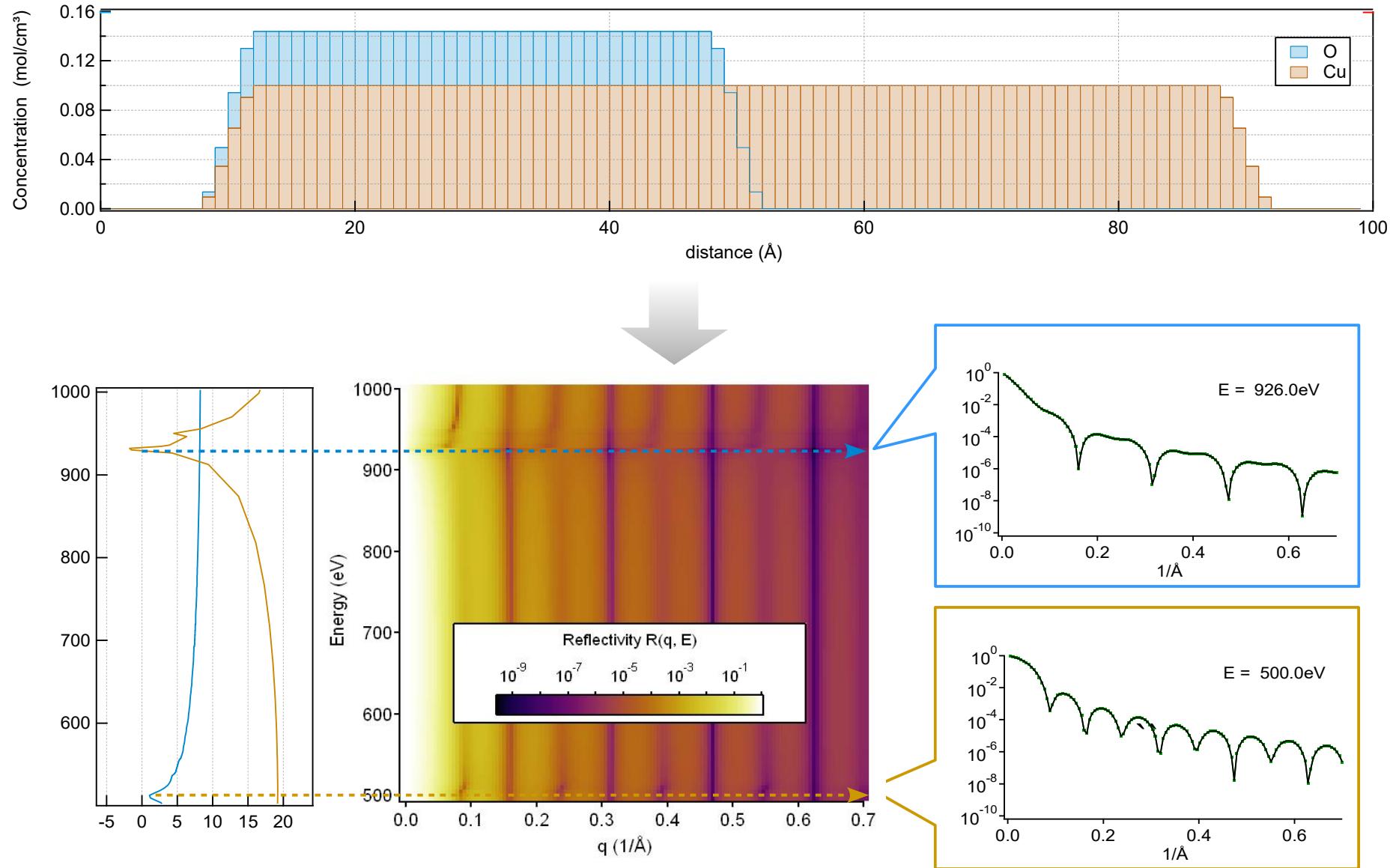
sum over chemical elements atomic concentration in mol/cm³

$$n-1 = \sum_a \frac{2\pi c_a r_0}{k^2} \underbrace{(f'(E) + if''(E))}_{\text{atomic scattering length}}$$

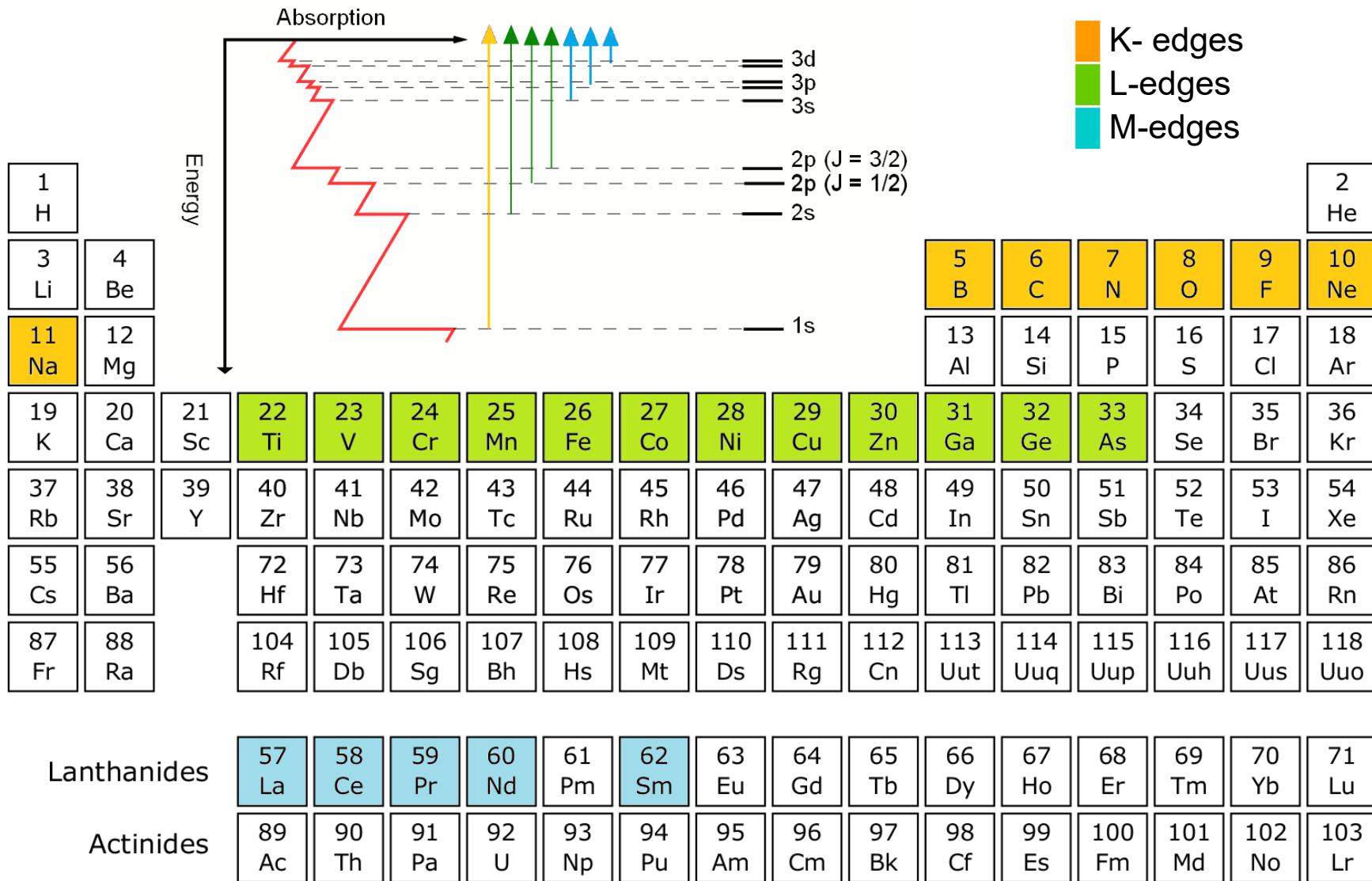


Lab-based of single-energy reflectivity measurements
cannot be element sensitive.

Where does element sensitivity arise from?



“Habitable zone,, for soft X-ray reflectivity



Optical constants

1. Theoretical calculation away from resonances

- Chantler et al. AIP **652**, 370 (2003)
- Chantler et al. J Phys Chem Ref Data **24**, 71 (1995)
- Chantler et al. J Phys Chem Ref Data **29**, 597 (2000)

Theoretical Form Factor, Attenuation and Scattering Tabulation
for $Z = 1\text{--}92$ from $E = 1\text{--}10$ eV to $E = 0.4\text{--}1.0$ MeV

C. T. Chantler

School of Physics, University of Melbourne, Parkville, Victoria 3052, Australia^a

Received September 27, 1994; revised manuscript received October 5, 1994

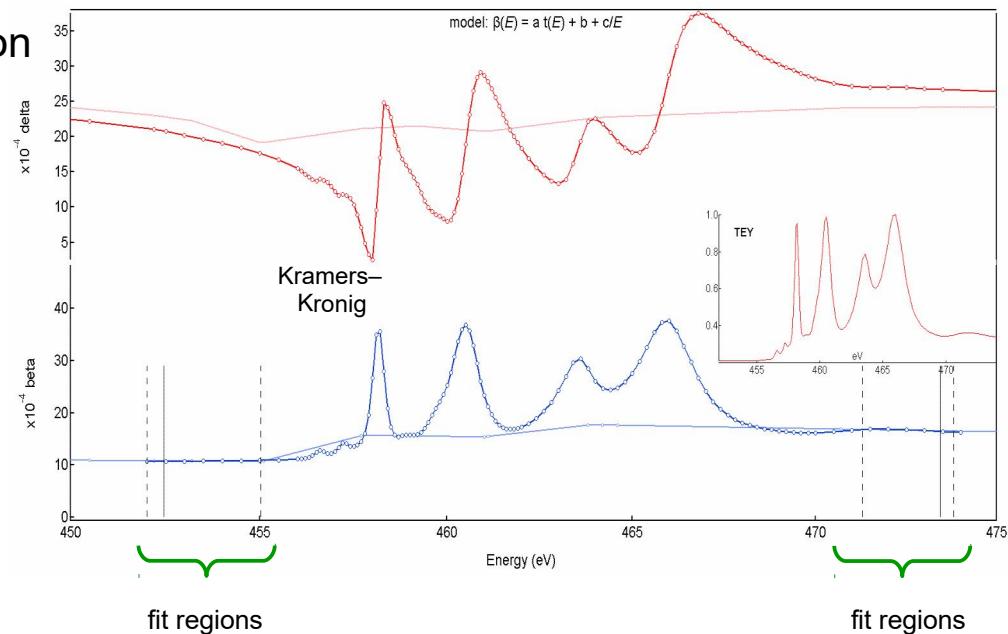
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THEORETICAL FORM FACTOR, ATTENUATION, AND SCATTERING TABULATION FOR $Z = 1\text{--}92$ 187

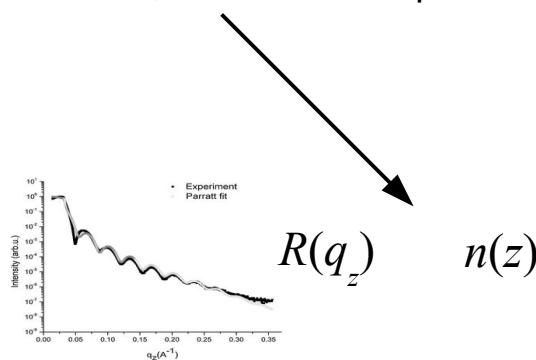
E keV	f_1 e/atom	f_2 e/atom	μ cm^2/g	$\sigma(\text{coh+inc})$ cm^2/g	μ_{sc} cm^2/g	λ nm
4.118862E+00	1.64400E+01	3.8997E+00	9.9405E+02	2.1733E+00	8.922E+02	3.010E-01
4.314821E+00	1.80399E+01	3.5640E+00	8.6321E+02	2.0998E+00	7.751E+02	2.860E-01
4.633924E+00	1.88984E+01	3.1991E+00	7.2483E+02	2.0037E+00	6.518E+02	2.676E-01
4.953664E+00	1.93754E+01	2.8820E+00	6.1083E+02	1.9080E+00	5.503E+02	2.503E-01
5.295467E+00	1.96982E+01	2.5972E+00	5.1493E+02	1.8133E+00	4.647E+02	2.341E-01
5.295467E+00	1.96982E+01	2.5972E+00	5.1493E+02	1.8133E+00	4.647E+02	2.341E-01

2. Total Electron Yield used as absorption

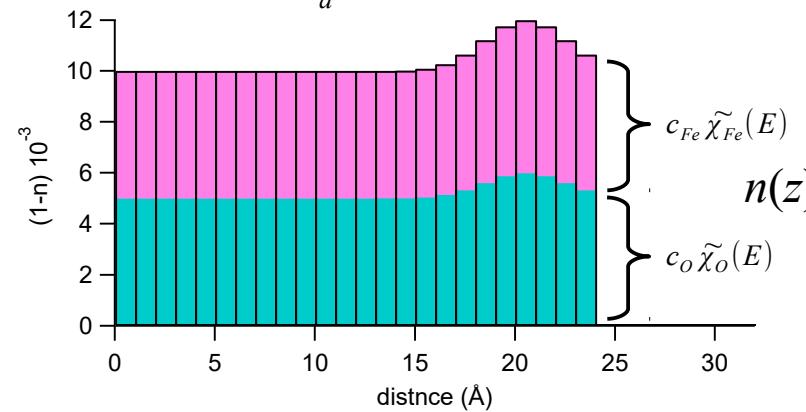


Where the vicious circle begin...

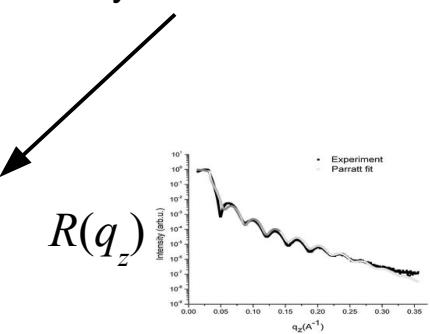
wanted, but close to impossible



$$n - 1 = \sum_a c_a \tilde{\chi}_a(E)$$



trivial and rarely needed



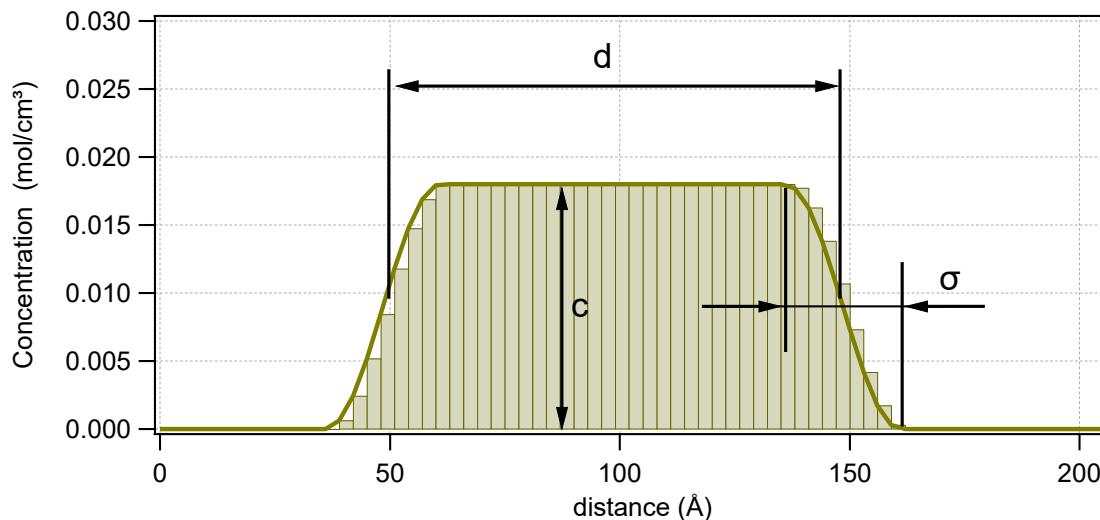
optimize & try again

Fitting R(q) to chemical profiles

Totally unrestricted fit:

- 1) 4 elements
- 2) 300Å thik sample (3Å x100 slices)
- 3) Elemental concentration in the range 0–0.100cm³ at Δc = 0.005cm³ resolution (5%)

Size of the “phase space” 400 fit parameters, each with 20 levels = $20^{400} \sim 2 \times 10^{520}$

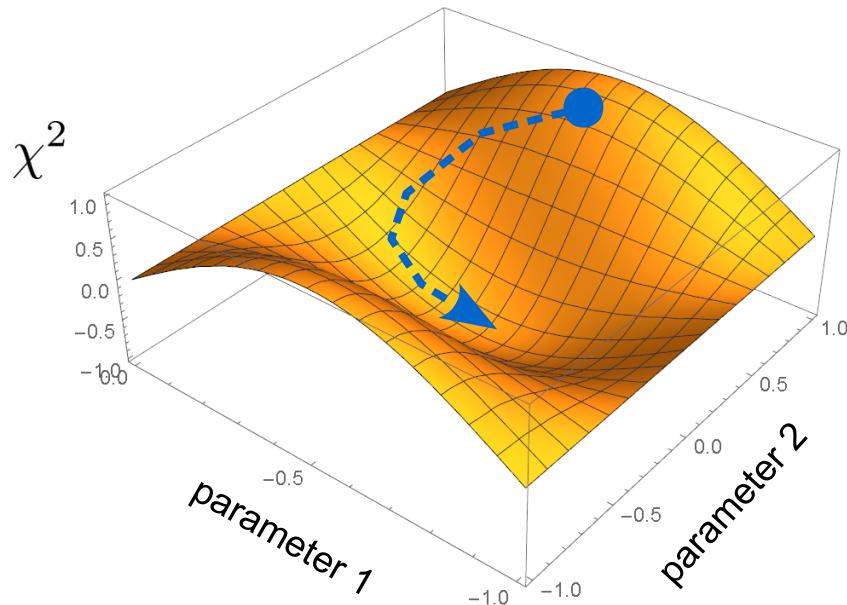


Parametrization in terms of Roughness σ , thickness d , concentration c , brings the number of “free” fit parameters typically below 3 x 3 layers x 4 elements = 36 $\Rightarrow 20^{36} \sim 7 \times 10^{46}$ distinct points in “phase space” with plenty of local minima.

\Rightarrow Necessity of sophisticated fit methods.

Fitness profile

$$\chi^2(p_1, \dots, p_N) = \sum_{\{q, \omega\}} [R_{\text{exp}}(q, \omega) - R_{\text{model}}(q, \omega, p_1, \dots, p_N)]^2 \rightarrow \min$$



Commonly used gradient descent methods are not suitable to fit reflectivity data, due to large number of local minima and strong non-linearity



Fitting approaches:

THE JOURNAL OF CHEMICAL PHYSICS **125**, 244702 (2006)

Fitting a free-form scattering length density profile to reflectivity data using temperature-proportional quenching

Charles F. Laub and Tonya L. Kuhl

Department of Chemical Engineering and Materials Science, University of California,
Davis, California 95616

(Received 31 May 2006; accepted 7 November 2006; published online 27 December 2006)

IOP PUBLISHING
J. Phys. D: Appl. Phys. **40** (2007) 6000–6004

JOURNAL OF PHYSICS D: APPLIED PHYSICS
doi:10.1088/0022-3727/40/19/033

Genetic algorithm using independent component analysis in x-ray reflectivity curve fitting of periodic layer structures

J. Phys. D: Appl. Phys. **33** (2000) 1757–1763. Printed in the UK

PII: S0022-3727(00)11144-1

Characterization of a layer stack by wavelet analysis on x-ray reflectivity data

E Smigiel and A Cornet

IOP PUBLISHING
J. Phys. D: Appl. Phys. **40** (2007) 6000–6004

JOURNAL OF PHYSICS D: APPLIED PHYSICS
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Genetic algorithm using independent component analysis in x-ray reflectivity curve fitting of periodic layer structures

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JOURNAL OF PHYSICS D: APPLIED PHYSICS
doi:10.1088/0022-3727/40/14/023

Fitness function and nonunique solutions in x-ray reflectivity curve fitting: crosserror between surface roughness and mass density

computer programs

Journal of
Applied
Crystallography
ISSN 0021-8898

Received 23 May 2007
Accepted 14 September 2007

GenX: an extensible X-ray reflectivity refinement program utilizing differential evolution

Matts Björck*† and Gabriella Andersson

Department of Physics, Uppsala Universitet, Box 530, SE-751 21, Uppsala, Sweden. Correspondence e-mail:
matts.bjorck@psi.ch

Some modification of an Evolution algorithms appear to be the most suited to fit reflectivity data.

Fitting R(q) to chemical profiles



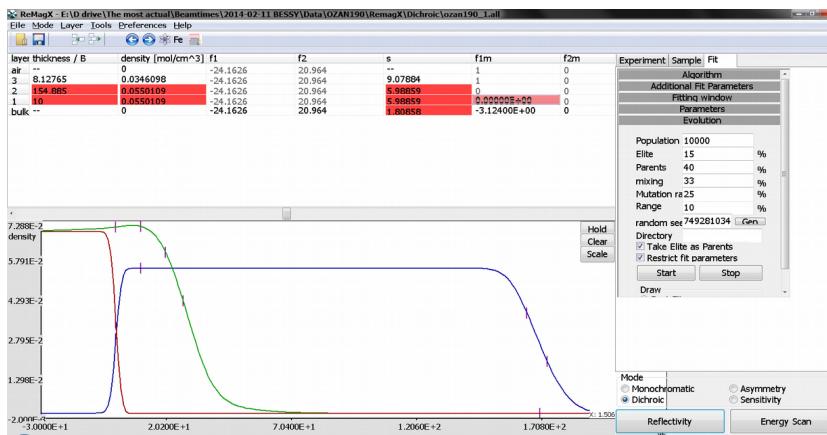
RemagX (S. Macke)

Simulation:

- Parrat formalism
- Matrix formalism

Fit:

- Evolution
- Simplex
- Levenberg&Marquardt



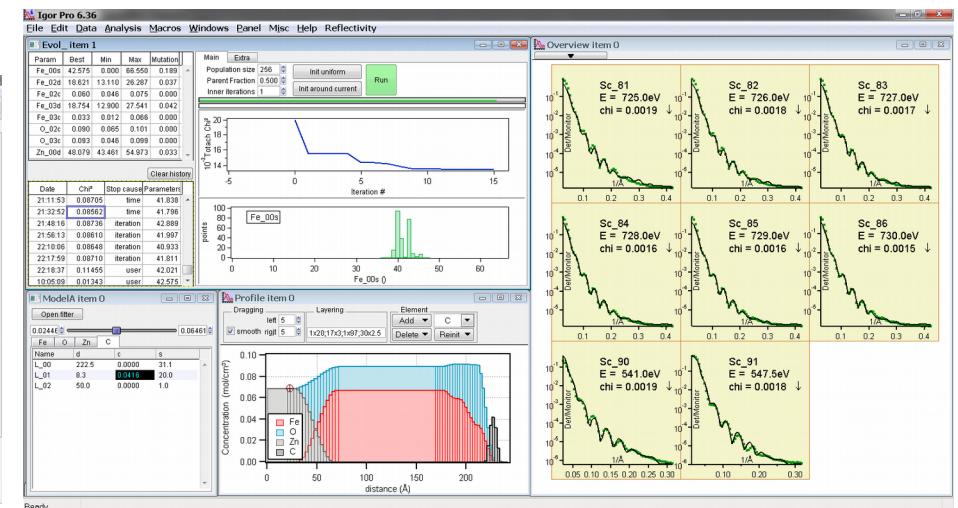
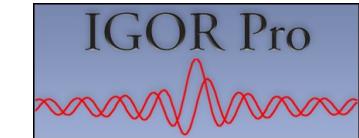
Own code under IgorPro

Simulation:

- Parrat formalism
- Coupling with theory (Quanty)

Fit:

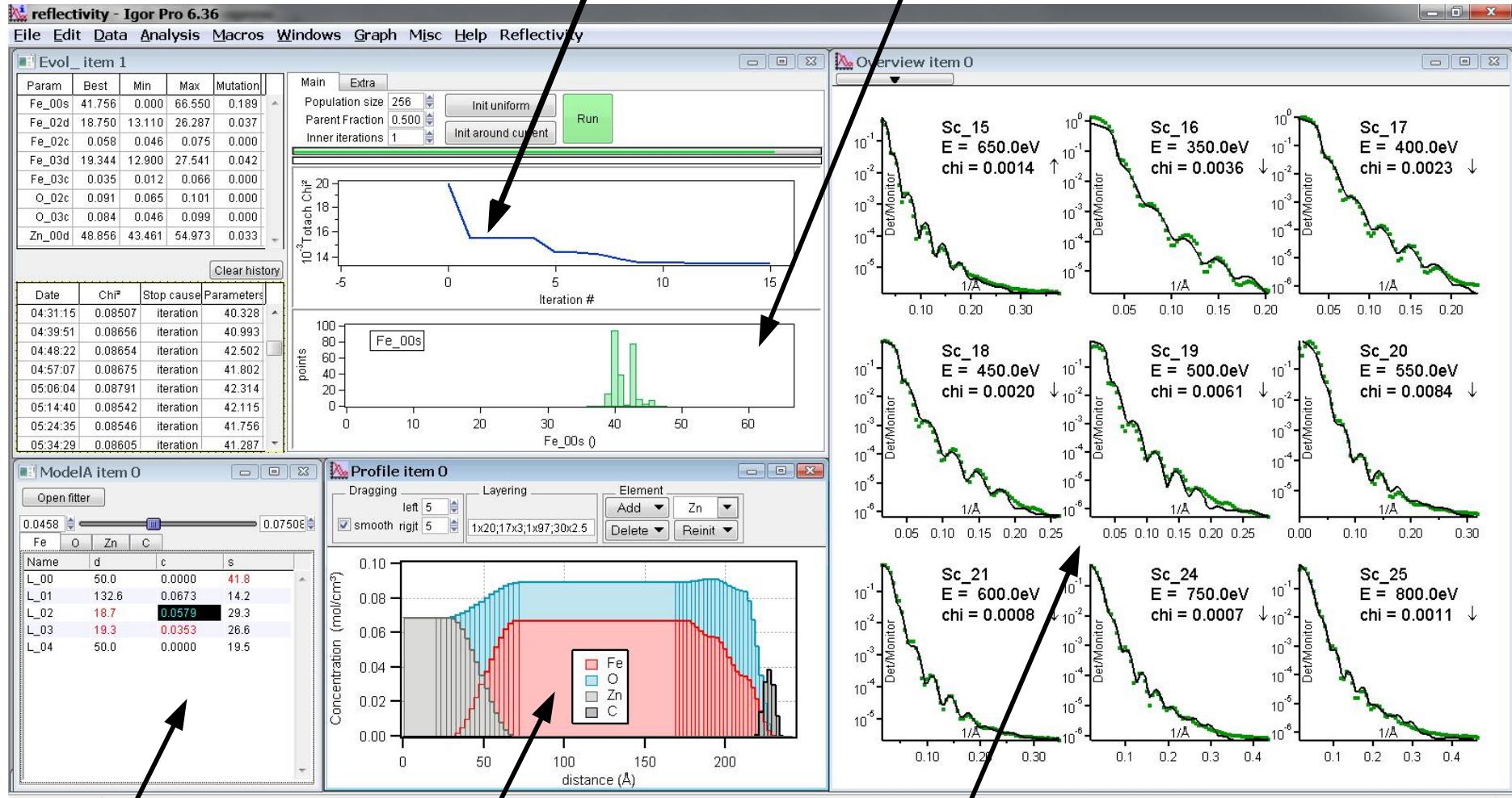
- Evolution+dynamic mutation
- direct import of SPECS data
- preprocessing (cutting, resampling, background)



Fitting R(q) to chemical profiles

Time convergence of $\chi^2(p_1, \dots p_N)$

Param. distribution in the population



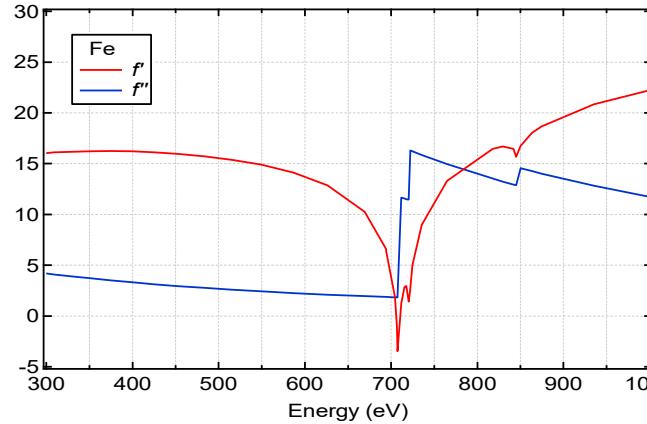
dσ-model

Atomic conc. profiles

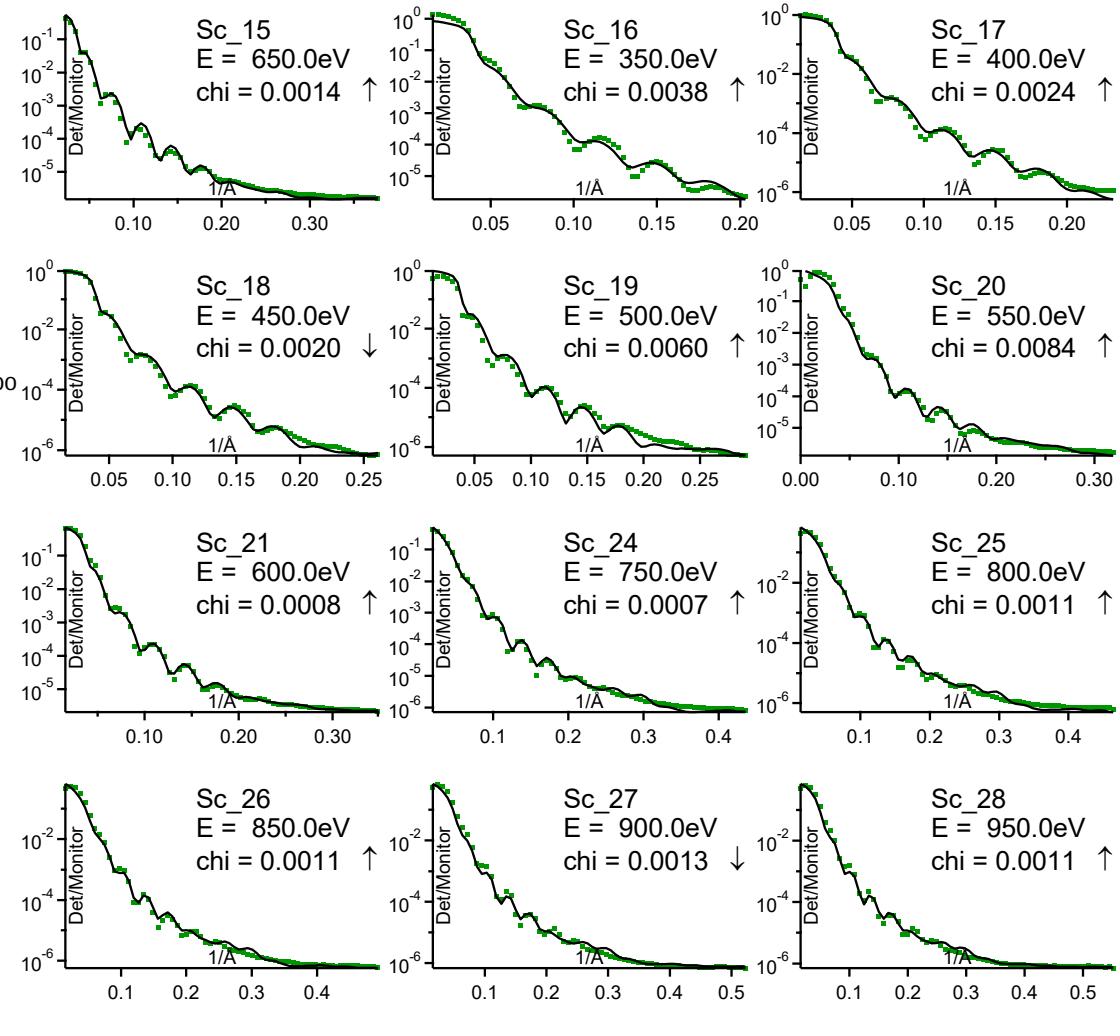
Overview of all experimental and simulated spectra

“Real life” example: ZnO/Fe₃O₄

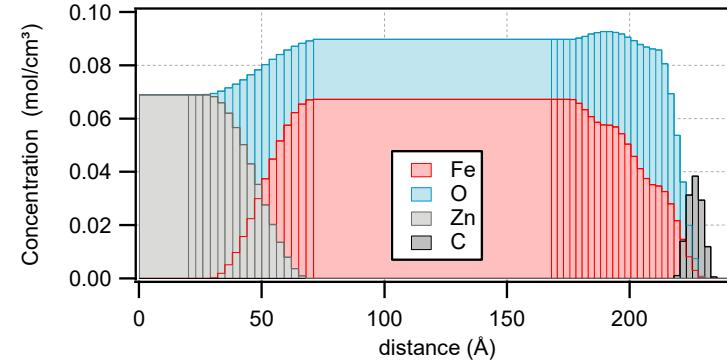
Fe optical constants, Chantler



Off-resonance spectra, E = 350–1000eV

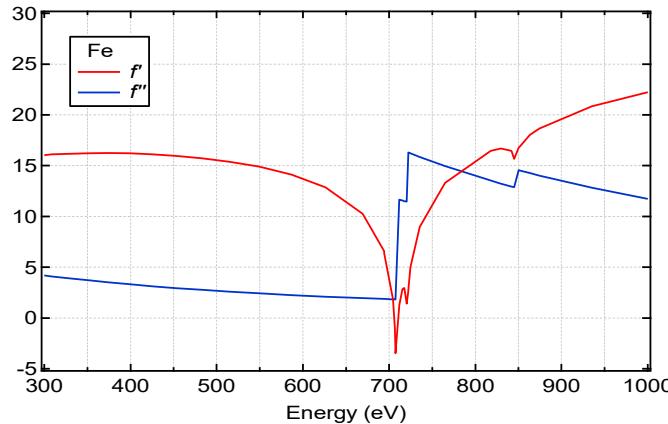


Chemical profile

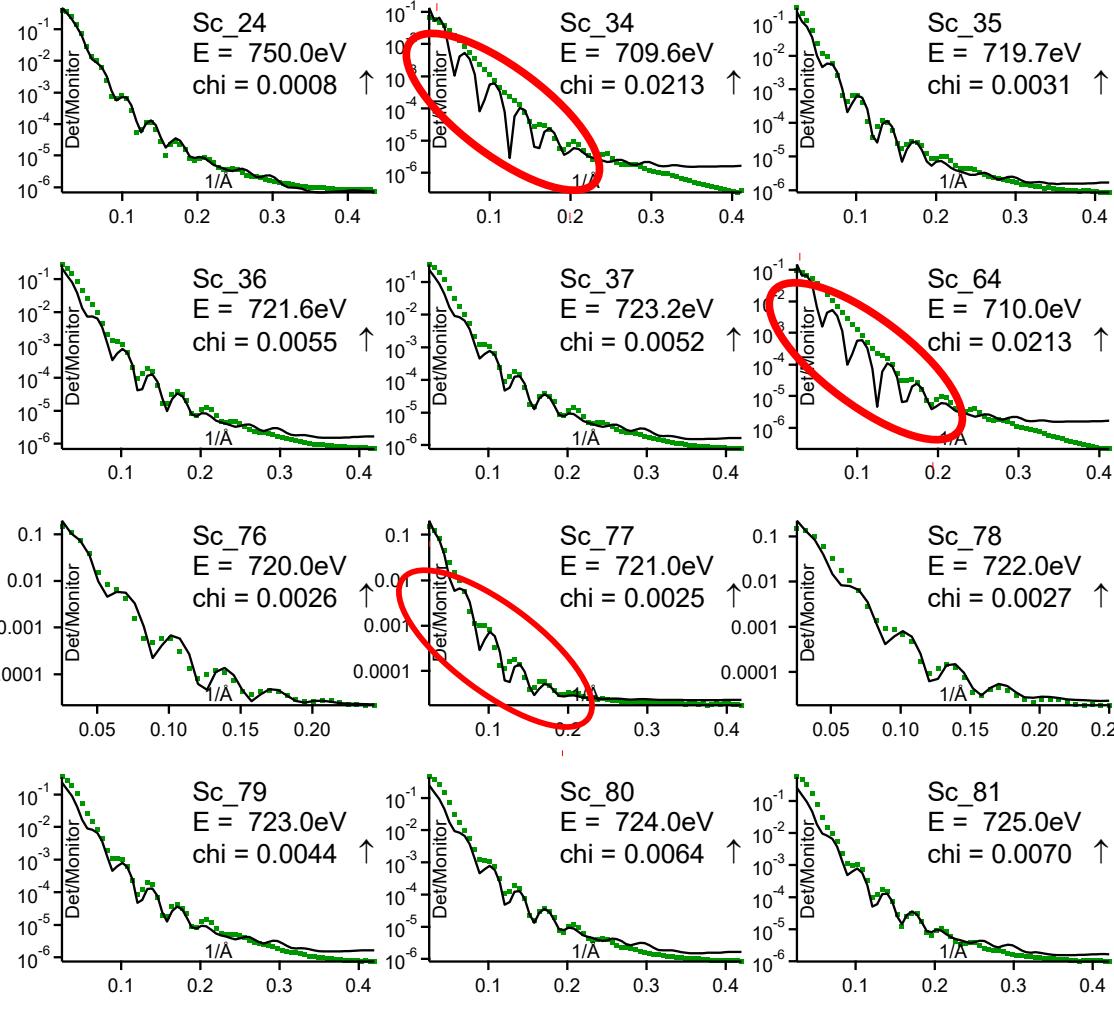


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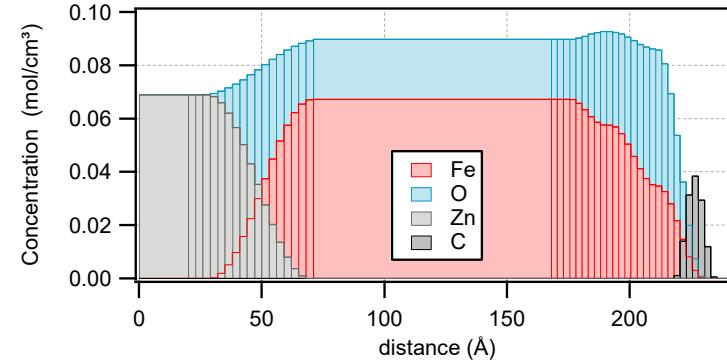
Fe optical constants, Chantler



on-resonance spectra, E = 700–730eV

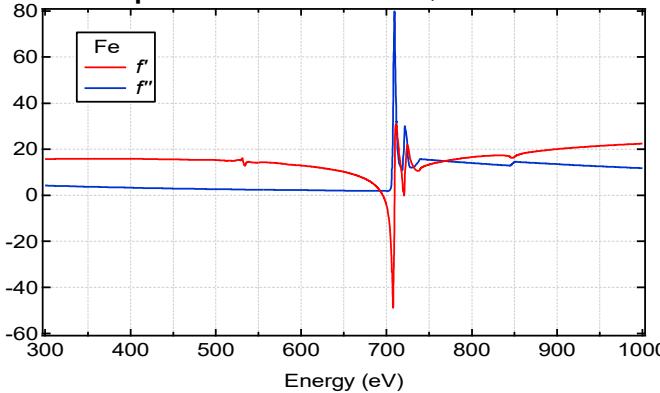


Chemical profile

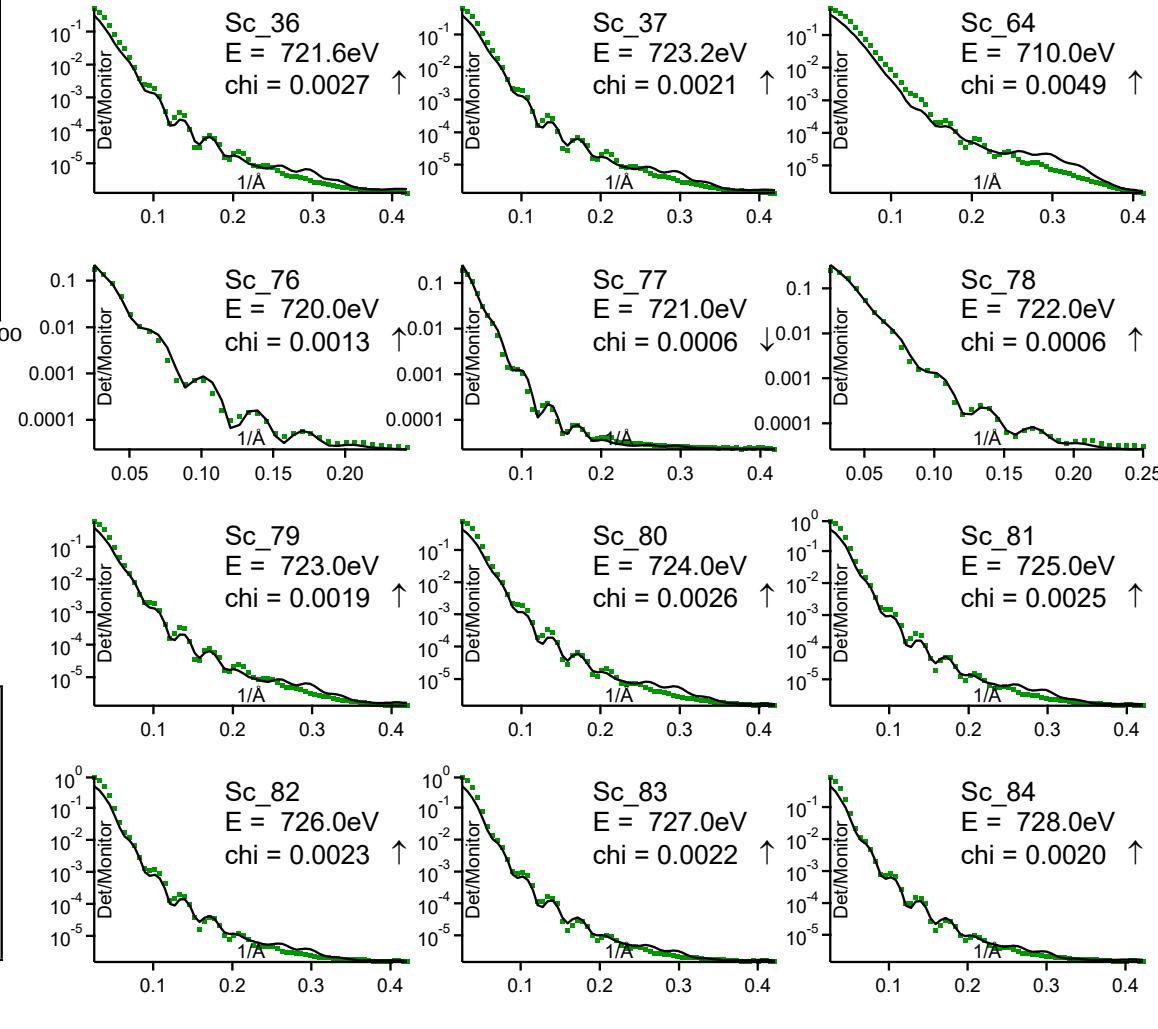


“Real life” example: ZnO/Fe₃O₄

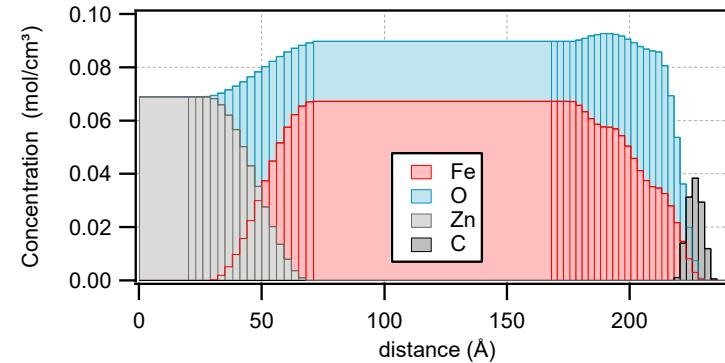
Fe optical constants, corrected



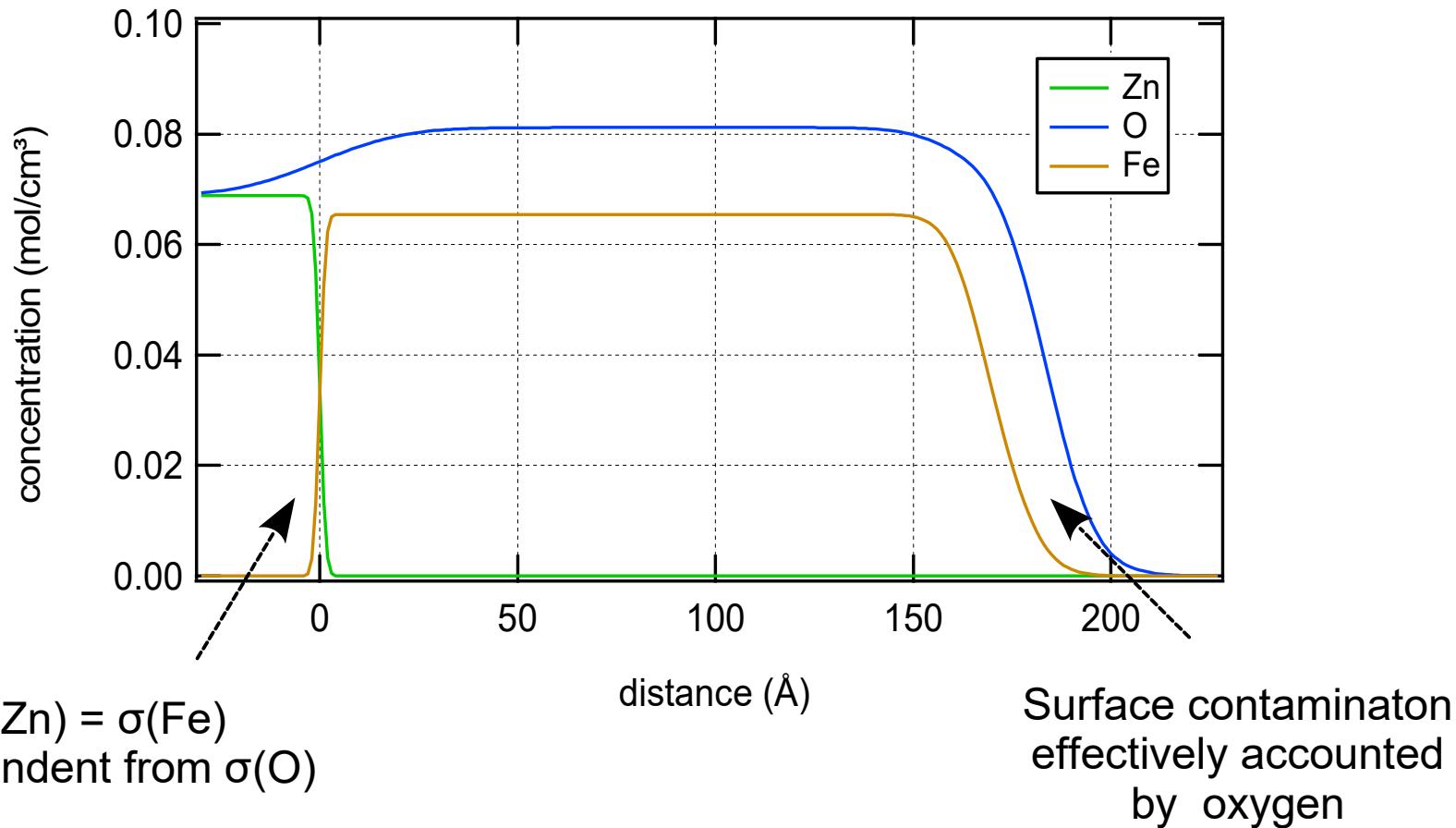
on-resonance spectra, E = 700–730 eV



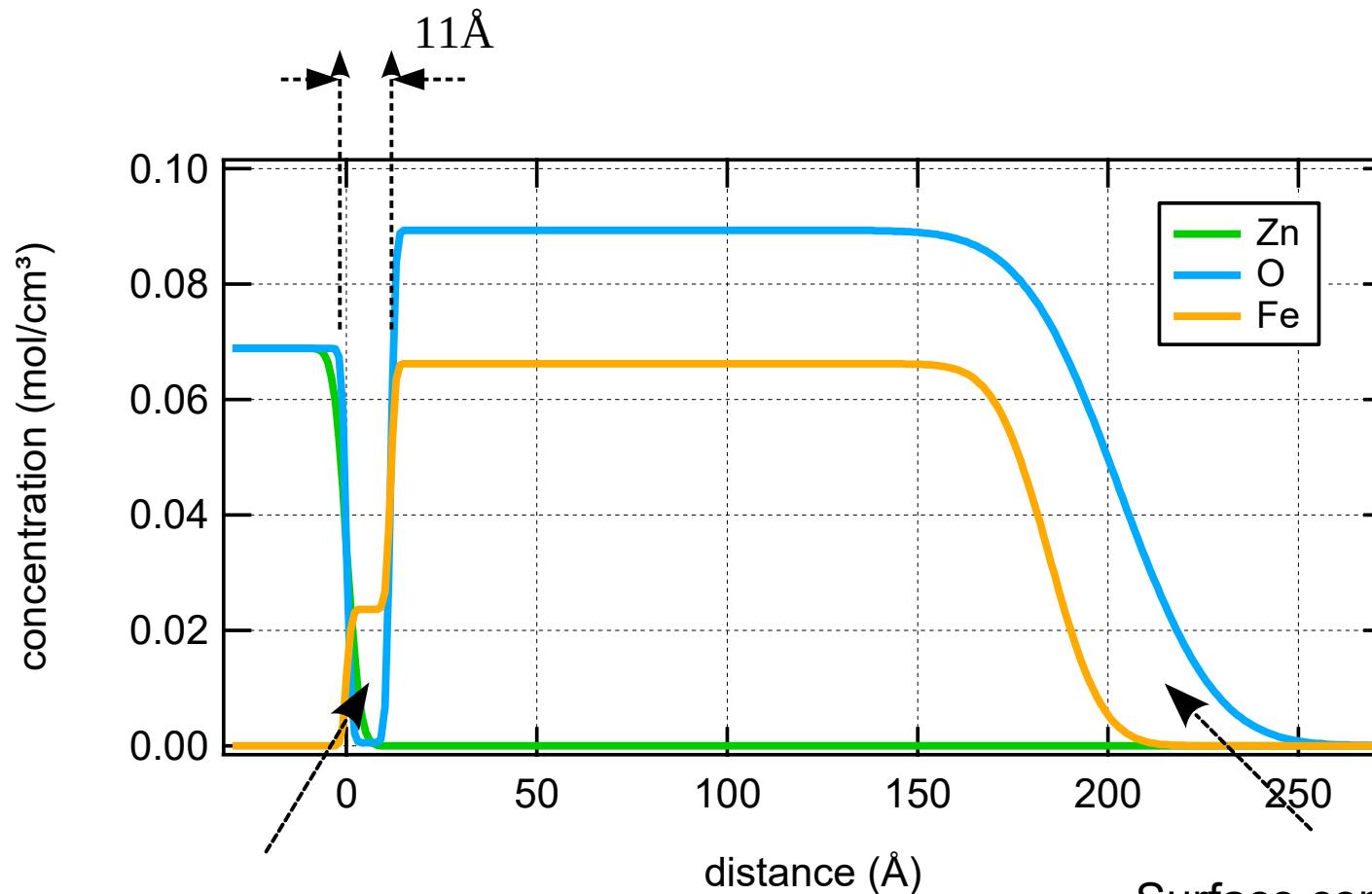
Chemical profile



Ozan190 – O terminated



Ozan194 – Zn terminated

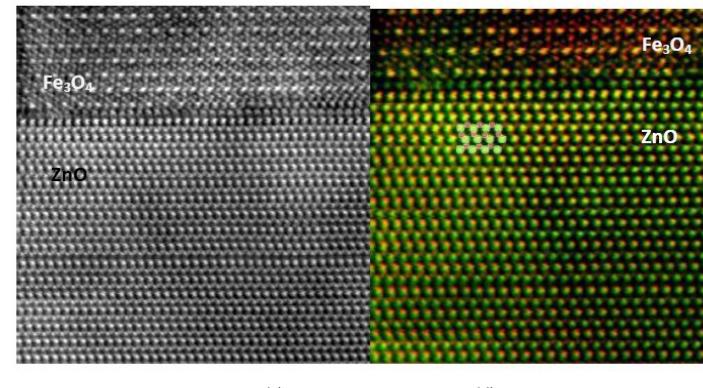
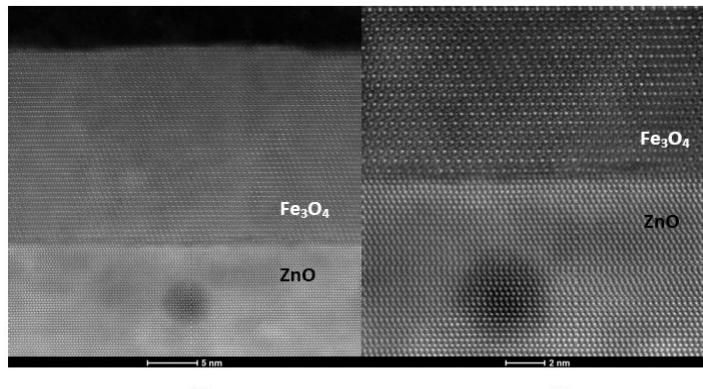


$\sigma(\text{Zn}) = \sigma(\text{Fe}) \text{ independent from } \sigma(\text{O})$

Surface contamination
effectively accounted
by oxygen

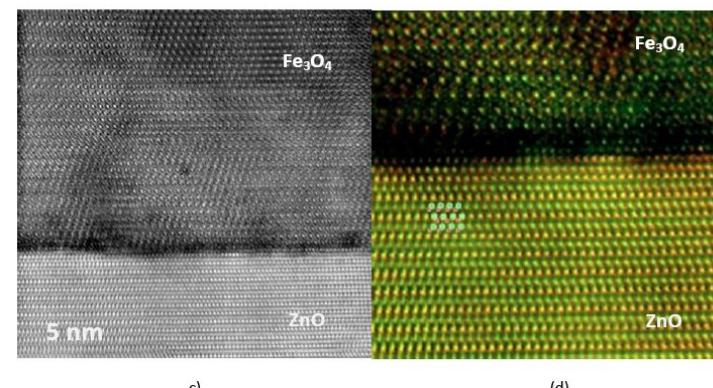
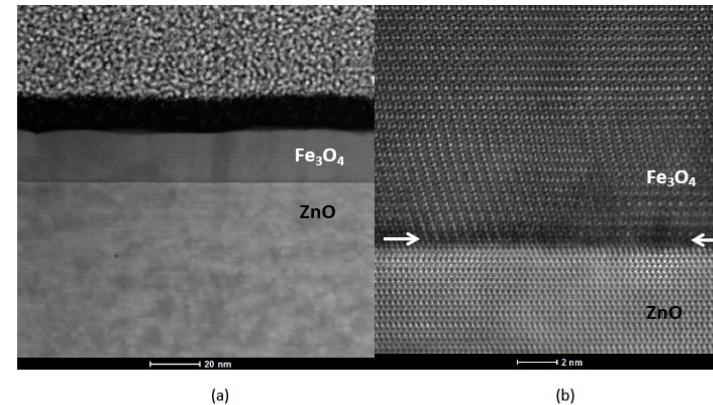
Comparison to STEM

O terminated



O. Kirilmaz et al. in preparation

Zn terminated



exfoliation?

Optical constants

1. Theoretical calculation away from resonances

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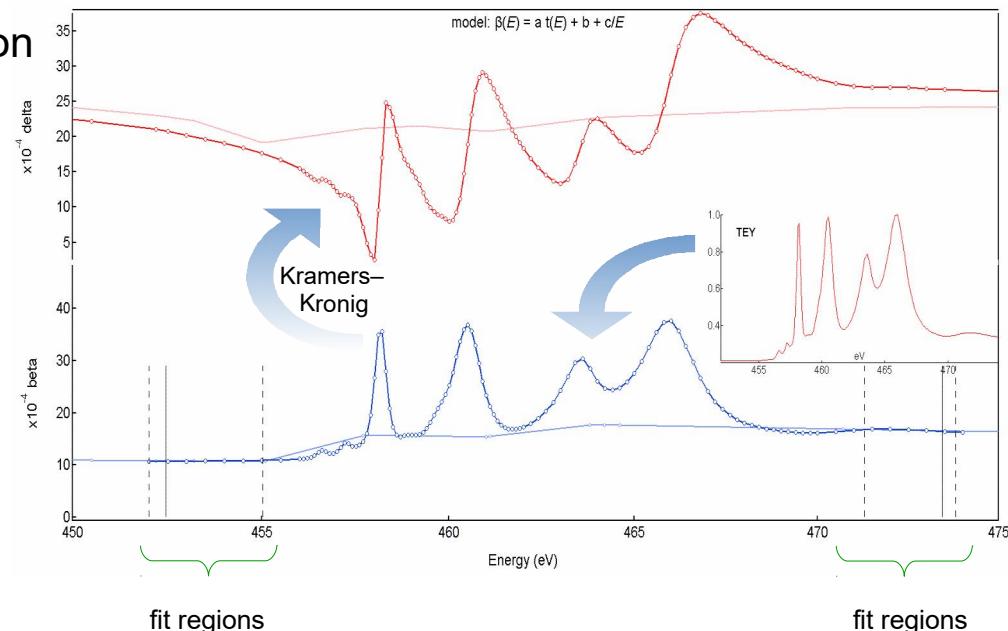
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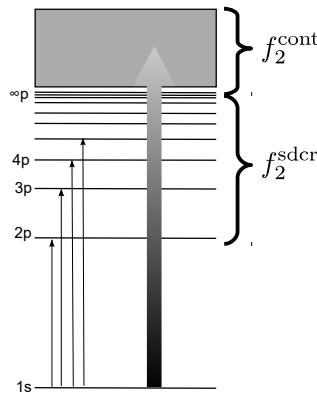
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4.314821E+00	1.80399E+01	3.5640E+00	8.6321E+02	2.0998E+00	7.751E+02	2.860E-01
4.633924E+00	1.88984E+01	3.1991E+00	7.2483E+02	2.0037E+00	6.518E+02	2.676E-01
4.953664E+00	1.93754E+01	2.8820E+00	6.1083E+02	1.9080E+00	5.503E+02	2.503E-01
5.295467E+00	1.96982E+01	2.5972E+00	5.1493E+02	1.8133E+00	4.647E+02	2.341E-01
5.295467E+00	1.96982E+01	2.5972E+00	5.1493E+02	1.8133E+00	4.647E+02	2.341E-01

2. Total Electron Yield used as absorption



Hydrogen atom

H atom is a suitable test system, since its discrete and continuous spectrum of is known analytically



$f_2 \rightarrow \sigma_{\text{abs}} \rightarrow$ transition rate \rightarrow matrix element

$$f_2(\hbar\omega) = \frac{\omega\sigma_{\text{abs}}}{4\pi c r_e} = \frac{\omega\sigma_{\text{abs}}}{4\pi c} \frac{4\pi\epsilon_0 mc^2}{e^2} = \frac{\omega\sigma_{\text{abs}}\epsilon_0 mc}{e^2} = \frac{2m\hbar}{A_z^2 e^2} \frac{dw}{dt}$$

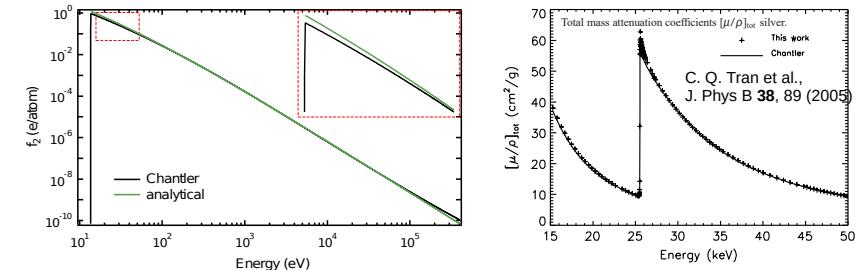
$$\frac{dw}{dt} = \frac{2\pi}{\hbar} \sum_{\mathbf{n}} |F_{\mathbf{n}_0, \mathbf{n}}|^2 \times \frac{1}{\pi} \text{Im} \left[\frac{1}{(E_{\mathbf{n}} - E_{\mathbf{n}_0} - \hbar\omega) - i\lambda} \right], \lambda \rightarrow 0$$

Analytical expressions for $f_2(E)$ can be derived:

$$f_2^{\text{cont}}(\hbar\omega) = \frac{2m\hbar}{A_z^2 e^2} \frac{dw}{dt} = \frac{64\pi}{3} \left(\frac{Ry}{\hbar\omega} \right)^3 \frac{1}{(e^{2\pi/\kappa_0} - 1)} \exp \left[\frac{4 \arctan(1/\kappa_0)}{\kappa_0} \right]$$

$$f_2^{\text{dscr}}(\hbar\omega) = \frac{128\pi}{3} \left(\frac{\hbar\omega}{Ry} \right)^2 \sum_{n=2}^{+\infty} \frac{1}{n^3} \left(\frac{n-1}{n+1} \right)^{2n} \frac{1}{\left(1 - \frac{1}{n^2} \right)^5} \times \frac{1}{\pi} \text{Im} \left[\frac{1}{\left(1 - \frac{1}{n^2} - \frac{\hbar\omega}{Ry} \right) + i\frac{\lambda}{Ry}} \right]$$

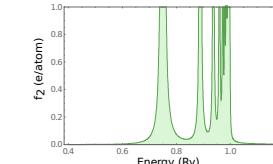
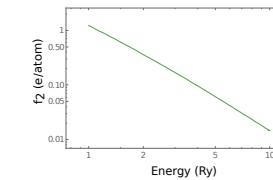
Comparison to C.T. Chanter data:



Comparison to experimental polarizability:

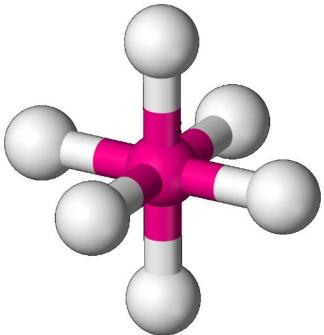
$$\alpha' = \lim_{\omega \rightarrow 0} \alpha'(\omega) = r_e c^2 \lim_{\omega \rightarrow 0} \frac{f_1(\omega)}{\omega^2}$$

$$\begin{aligned} \alpha'_{\text{tot}} &= \alpha'_{\text{dscr}} + \alpha'_{\text{cont}} = (5.424 + 1.239) \cdot 10^{-31} \approx \\ &\approx 6.66 \cdot 10^{-31} \text{ m}^3 \\ \alpha'_{\text{exp}} &= 6.67 \cdot 10^{-31} \text{ m}^3 \end{aligned}$$



Theoretical optical constants

Crystal/Ligand field cluster calculation



$$\hat{H}_{CM} = \sum_{\mu\sigma} \varepsilon_{\mu\sigma} \hat{l}_{\mu\sigma}^\dagger \hat{l}_{\mu\sigma} + \sum_{m\sigma} \varepsilon_{m\sigma} \hat{c}_{m\sigma}^\dagger \hat{c}_{m\sigma} + \sum_{n\sigma} \varepsilon_{n\sigma}^{2p} \hat{c}_{n\sigma}^\dagger \hat{c}_{n\sigma}$$

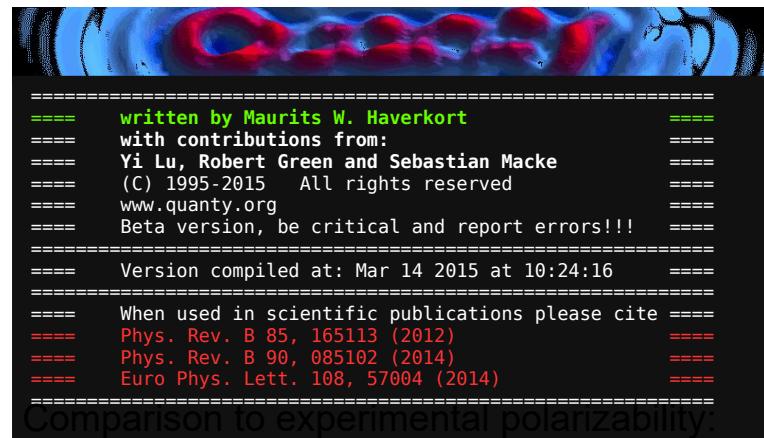
$$+ \sum_{\mu m \sigma} \left(t_{m\mu} \hat{c}_{m\sigma}^\dagger \hat{l}_{\mu\sigma} + t_{\mu m} \hat{l}_{\mu\sigma}^\dagger \hat{c}_{m\sigma} \right)$$

$$+ \frac{1}{2} \sum_{mm'm''m'''} \sum_{\sigma,\sigma'} U_{mm'm''m'''}^{3d-3d} \hat{c}_{m\sigma}^\dagger \hat{c}_{m''\sigma'}^\dagger \hat{c}_{m''' \sigma} \hat{c}_{m'''\sigma'}$$

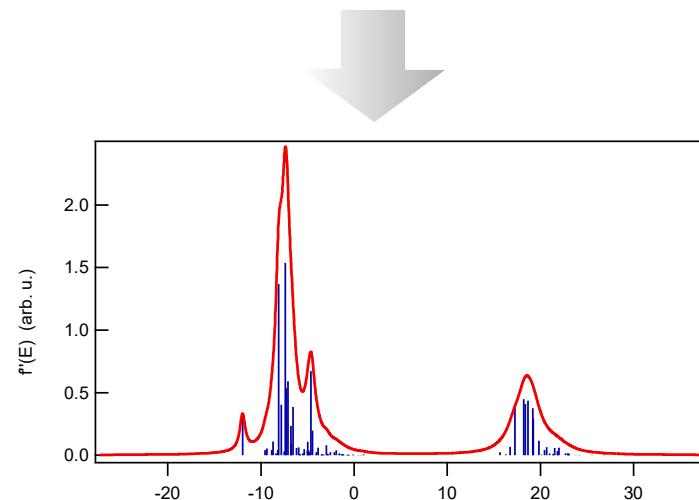
$$+ \sum_{mm'nn'} \sum_{\sigma,\sigma'} U_{mn m'n'}^{3d-2p} \hat{c}_{m\sigma}^\dagger \hat{c}_{n\sigma'}^\dagger \hat{c}_{n'\sigma} \hat{c}_{m'\sigma'}$$

$$+ \frac{\lambda_{SO}^{3d}}{2} \sum_{mm',\sigma\sigma'} \left[\hat{L} \cdot \hat{S} \right]_{m\sigma,m'\sigma'} \hat{c}_{m\sigma}^\dagger \hat{c}_{m'\sigma'}$$

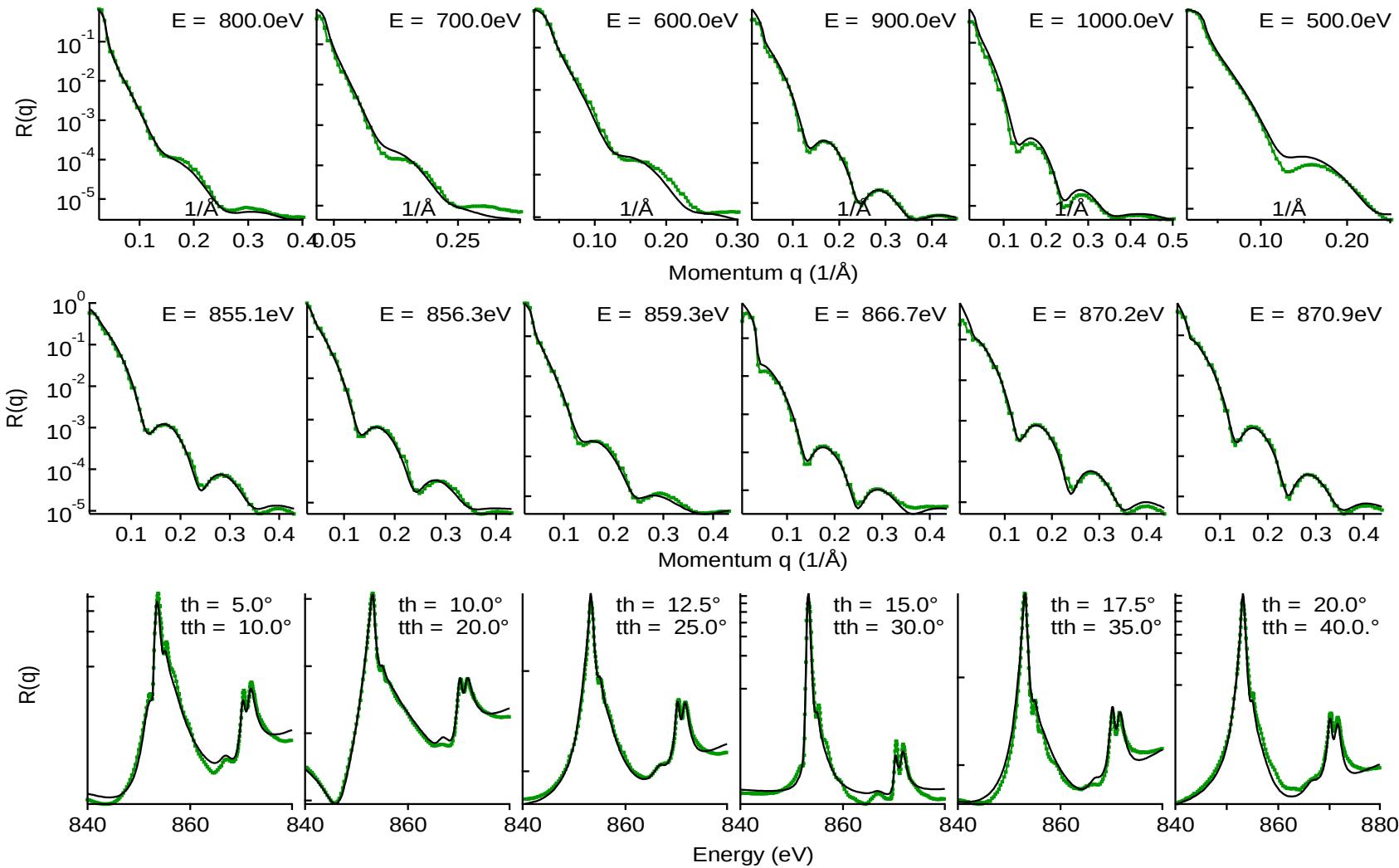
$$+ \frac{\lambda_{SO}^{2p}}{2} \sum_{nn',\sigma\sigma'} \left[\hat{L} \cdot \hat{S} \right]_{n\sigma,n'\sigma'} \hat{c}_{n\sigma}^\dagger \hat{c}_{n'\sigma'}$$



$$f_2^{\text{res}}(E) \sim \sum_f |\langle \Psi_i | \cos \theta | \Psi_f \rangle|^2 \text{Im} \left[\frac{1}{(E_f - E_i - E) - i\Gamma} \right]$$

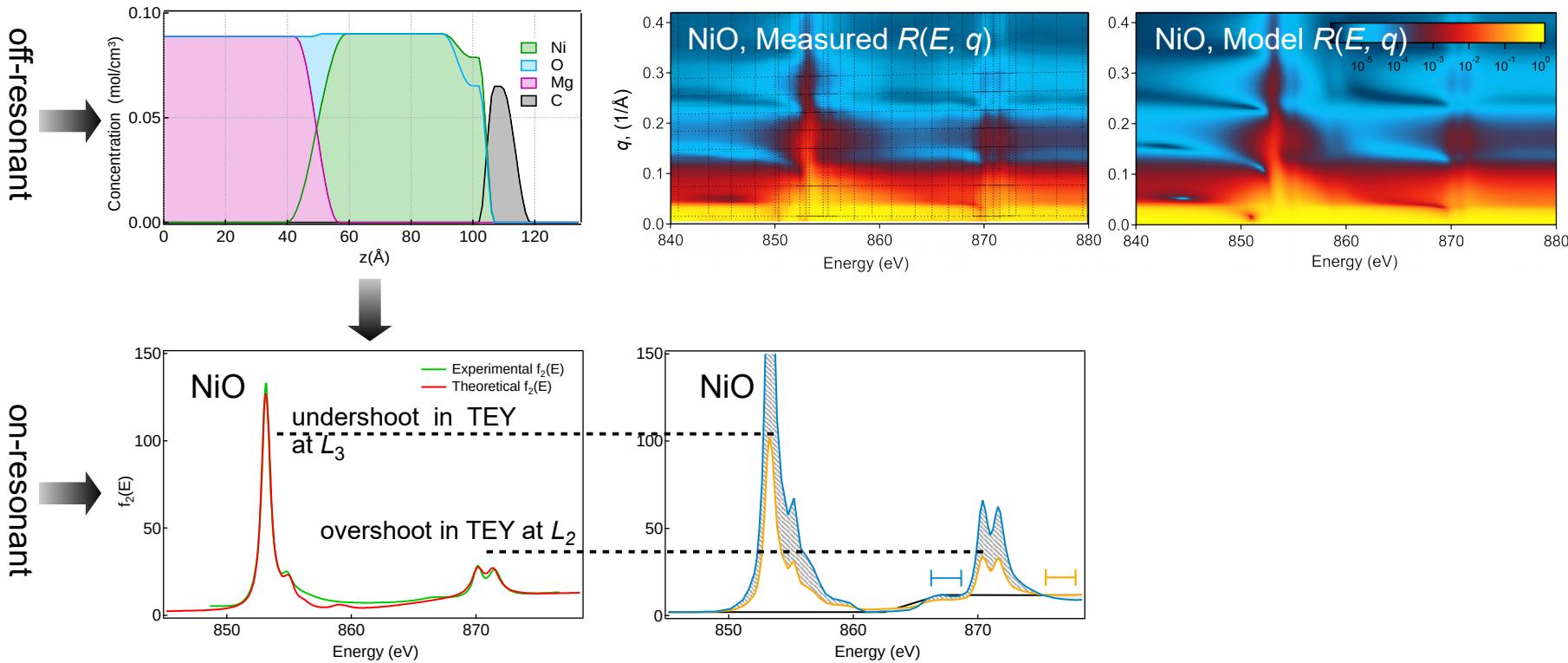


NiO – Theory restricted fit

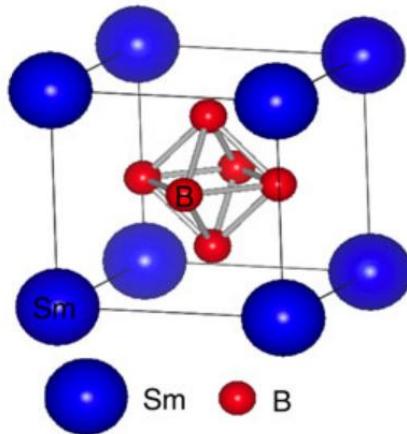


NiO – Theory restricted fit

$$f_2^{\text{res}}(E) = \boxed{\frac{mE^2}{\hbar^2} |\langle R_{2p}|r|R_{3d}\rangle|^2} \sum_f |\langle \Psi_i | \cos \theta | \Psi_f \rangle|^2 \text{Im} \left[\frac{1}{(E_f - E_i - E) - i\Gamma} \right].$$

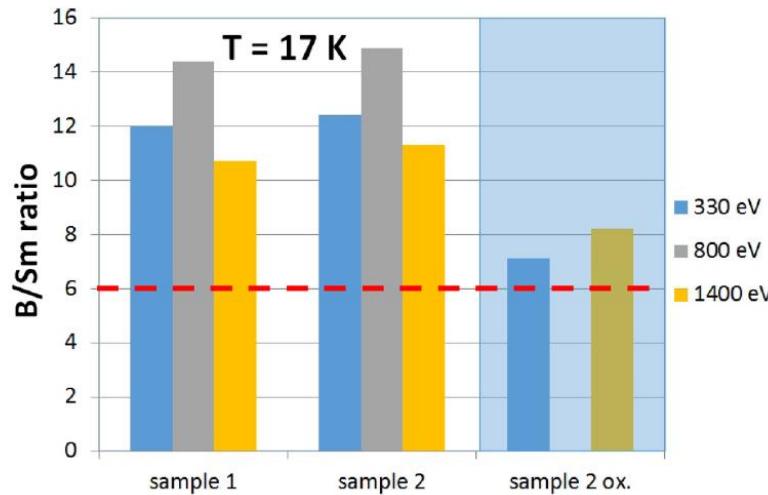


	10Dq	T_{pp}	ζ_{3d}	$F_{dd}^{(2)}$	$F_{dd}^{(4)}$	ζ_{2p}	$F_{2p3d}^{(2)}$	$G_{2p3d}^{(1)}$	$G_{2p3d}^{(3)}$	V_{eg}	V_{t2g}
NiO, starting values	0.56	0.72	0.08	11.14	6.87	11.51	6.67	4.92	2.80	2.06	1.21
fitted values	0.50	0.74	0.10	12.51	7.72	11.32	7.49	5.53	3.15	1.88	1.26



- Mixed valent system
- Kondo insulator
- Topologically protected ingap surface state

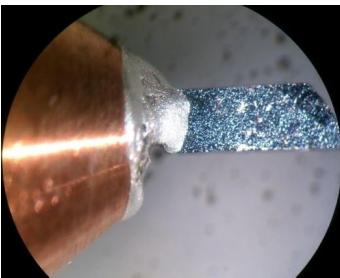
M. Neupane et al. Nat Comm 4, 2991 (2013)



Disrupted stoichiometry at the cleaved surface?

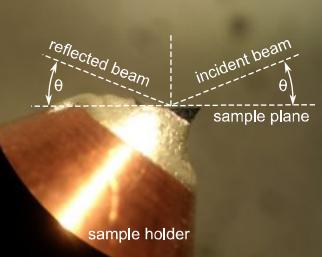
Heming et al. B 90, 195128 (2014)

SmB_6 – samples



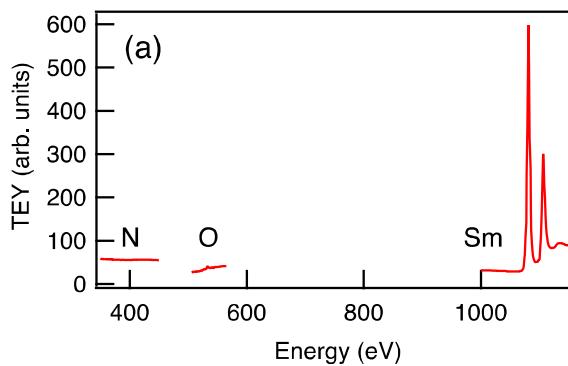
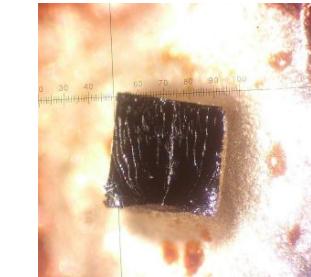
Often samples with polished surfaces are used for resonant x-ray experiments

- Surface contaminated and unstable
- It is too rough for reflectivity



We use floating-zone samples, cut in rods, pre-oriented

- Cleavage in fast entry at 10^{-8} mBar, transfer to 2×10^{-10} mBar in reflectivity chamber
- Cleaves along (100) even if cut along (111)
- Surface size $\sim 1\text{mm} \times 1\text{mm}$
- Sufficiently large terraces (beam $\sim 100\text{ }\mu\text{m}$)



No indications of oxygen in TEY spectrum

SmB₆ – a multivalent system

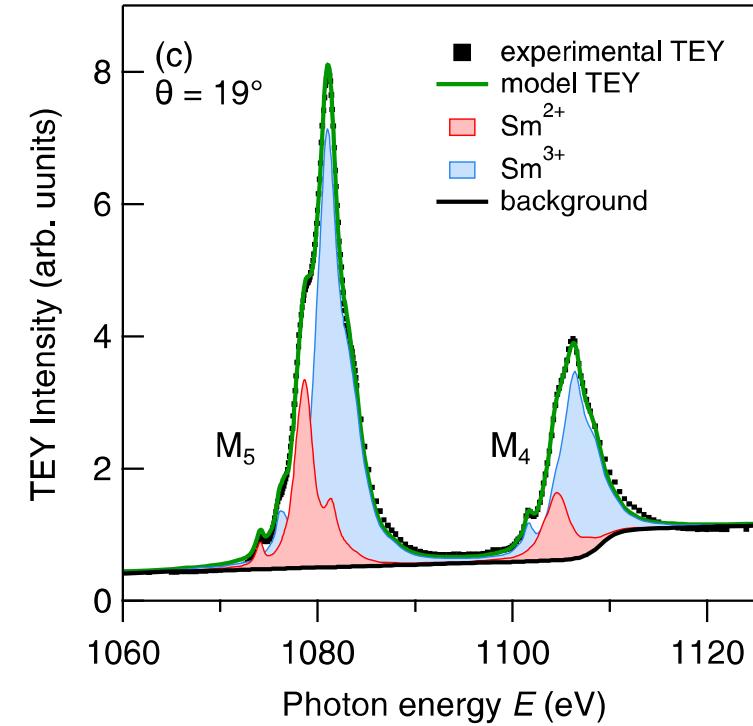
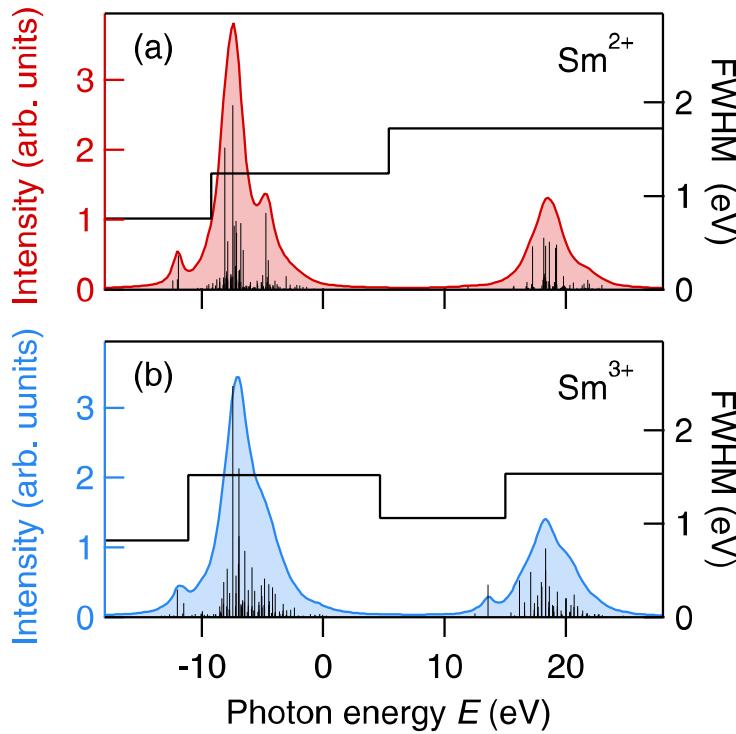
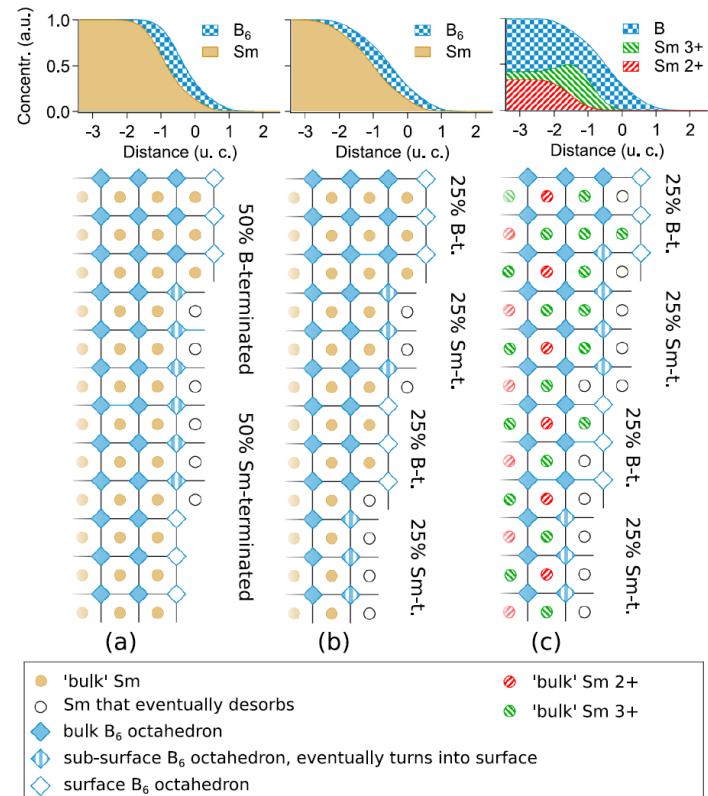
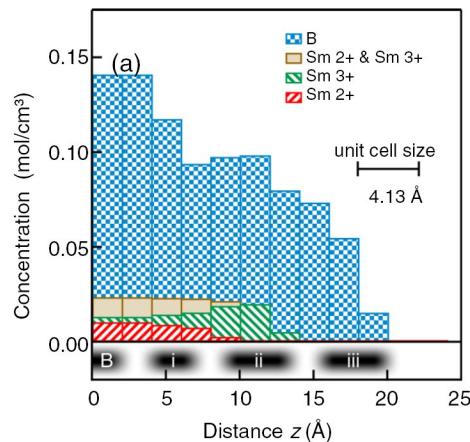
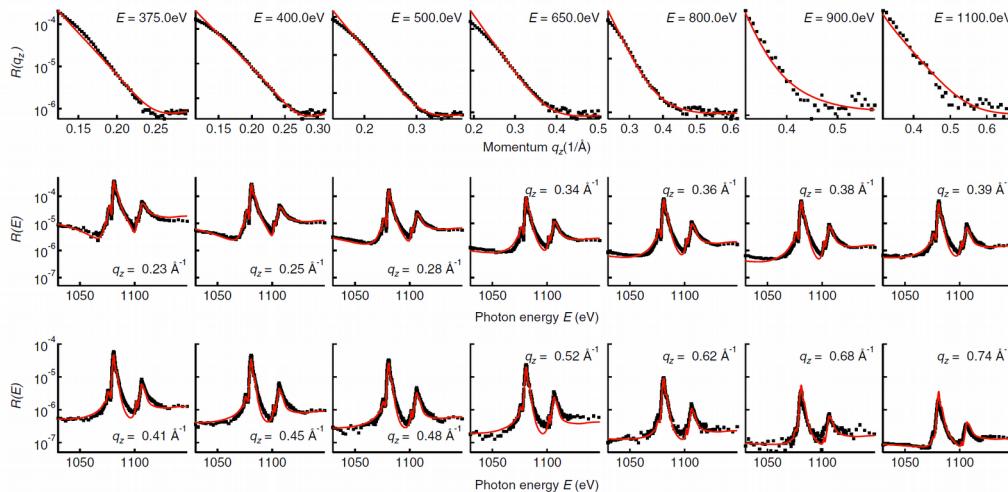


TABLE I. Optimized CFT parameters for Sm^{2+} and Sm^{3+} ions.

Ion	State	Configuration	$F_{ff}^{(2)}$	$F_{ff}^{(4)}$	$F_{ff}^{(6)}$	ζ_{4f}	$F_{df}^{(2)}$	$F_{df}^{(4)}$	$G_{df}^{(1)}$	$G_{df}^{(3)}$	$G_{df}^{(5)}$	ζ_{3d}
Sm^{2+}	Initial	$3d^{10}4f^6$	10.828	6.751	4.845	0.136						
	Final	$3d^94f^7$	11.548	7.218	5.185	0.165	6.701	3.075	4.670	2.734	1.888	10.514
Sm^{3+}	Initial	$3d^{10}4f^5$	10.950	6.873	4.945	0.152						
	Final	$3d^94f^6$	11.548	7.260	5.227	0.180	7.211	3.337	5.086	2.979	2.058	10.510

SmB₆ – a multivalent system



*Thank you for your attention
Questions are welcome.*



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