The axion electromagnetic response of topological insulators

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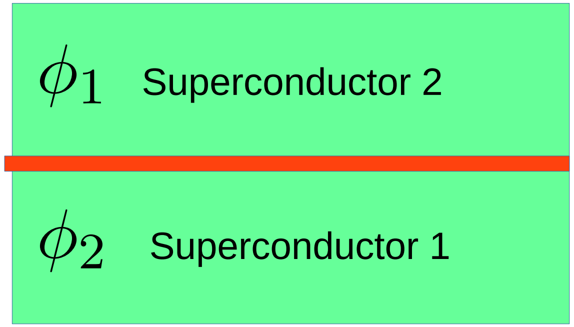
UKRATOP
IFW Dresden, 4.12.2018

Marriage of Josephson and Witten effect

Josephson effect

current due to phase difference

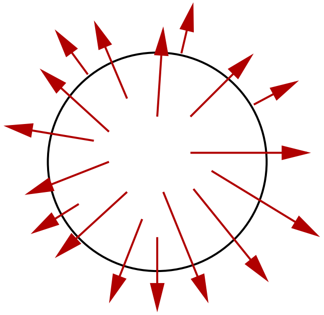
$$\Delta\phi = \phi_2 - \phi_1$$
$$I_J = I_c \sin(\Delta\phi)$$
$$\partial_t \Delta\phi = 2eV$$



Witten effect

charge fractionalisation due to magnetic monopoles

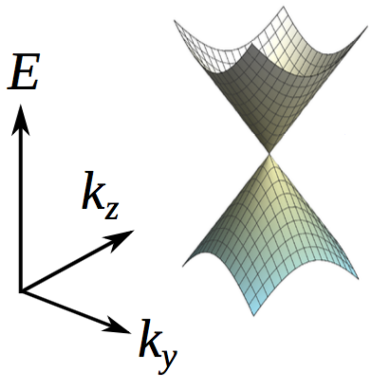
$$Q = e\left(n - \frac{\theta}{2\pi}\right)$$
$$\mathcal{L}_{Axion} = \frac{e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$



3D topological insulator

3D TI: insulating in bulk, conducting Dirac cone on surface

How do Dirac electrons couple to electromagnetic field?



More precisely: what is the effective EM Lagrangian $\mathcal{L}$ after integrating out electrons?

Maxwell: $$\mathcal{L}_M = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) - \rho\phi - \mathbf{j}\cdot\mathbf{A}$$

Answer: $$\mathcal{L} = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B} - \rho\phi - \mathbf{j}\cdot\mathbf{A}$$

Axion or θ term

Origin of θ-term

θ is given by the momentum space Chern-Simons form

$$\theta = \frac{1}{4\pi}\int d^3k\ \epsilon^{ijk}\ \mathrm{Tr}\ \left[\mathcal{A}_\mathrm{i}\partial_\mathrm{j}\mathcal{A}_\mathrm{k} - \mathrm{i}\frac{2}{3}\mathcal{A}_\mathrm{i}\mathcal{A}_\mathrm{j}\mathcal{A}_\mathrm{k}\right]$$

with the non-Abelian Berry vector potential

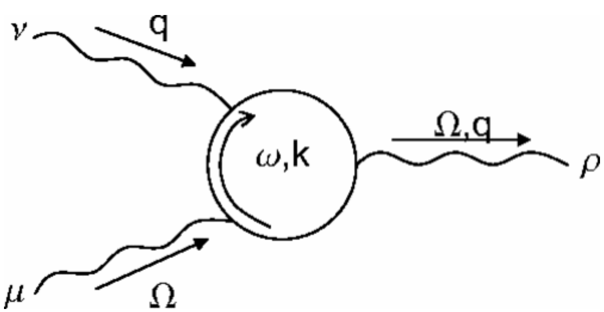
$$\mathcal{A}_i^{\alpha\beta}(\mathbf{k}) = -i\langle u_\mathbf{k}^\alpha|\partial_i|u_\mathbf{k}^\beta\rangle$$

and

$$\partial_i = \frac{\partial}{\partial k_i}$$

Qi, Hughes & Zhang, PRB 78, 195424 (2008)

For a time reversal invariant system θ either 0 or π



trivial insulator

3D TI

Consequence of θ-term: Witten effect

work out axion electrodynamics with Lagrangian…

$$\mathcal{L} = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B} - \rho\phi - \mathbf{j}\cdot\mathbf{A}$$

$$\mathbf{D} = \frac{\partial\mathcal{L}}{\partial\mathbf{E}} \qquad \mathbf{H} = -\frac{\partial\mathcal{L}}{\partial\mathbf{B}}$$

allow for magnetic monopoles…

$$\nabla\cdot\mathbf{D} = \rho \qquad \nabla\cdot\mathbf{B} = \rho_m \quad \rightarrow \nabla\cdot\mathbf{E} = 4\pi\left(\rho - \frac{e^2\theta}{4\pi^2}\rho_m\right)$$

… so that the total charge Q is

$$Q = \underset{\displaystyle\downarrow\atop\displaystyle ne}{q} - \frac{e^2\theta}{4\pi^2}\underset{\displaystyle\downarrow\atop\displaystyle 2\pi/e}{e_m} = e\left(n - \frac{\theta}{2\pi}\right)$$

Magnetic monopole fractionalizes electric charge !!

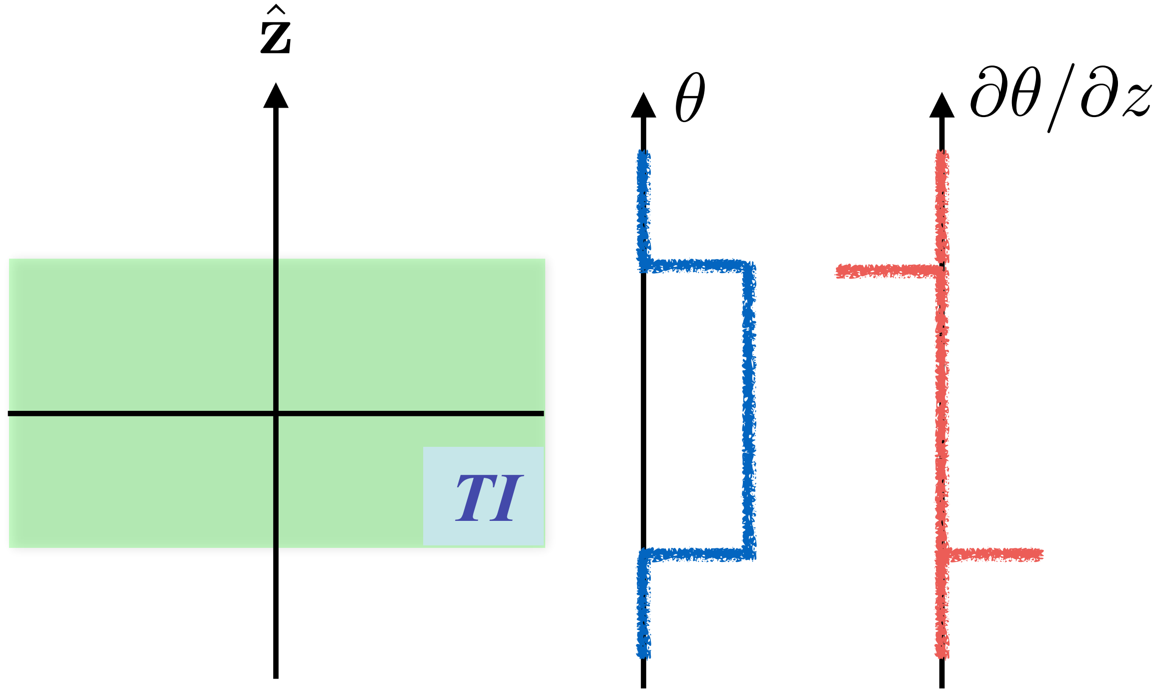
Witten, Phys. Lett. B 86, 283 (1979)

Uniform θ

in this case the axion term in the action is a total derivative

for an infinite 3D TI, Maxwell equations do not modify

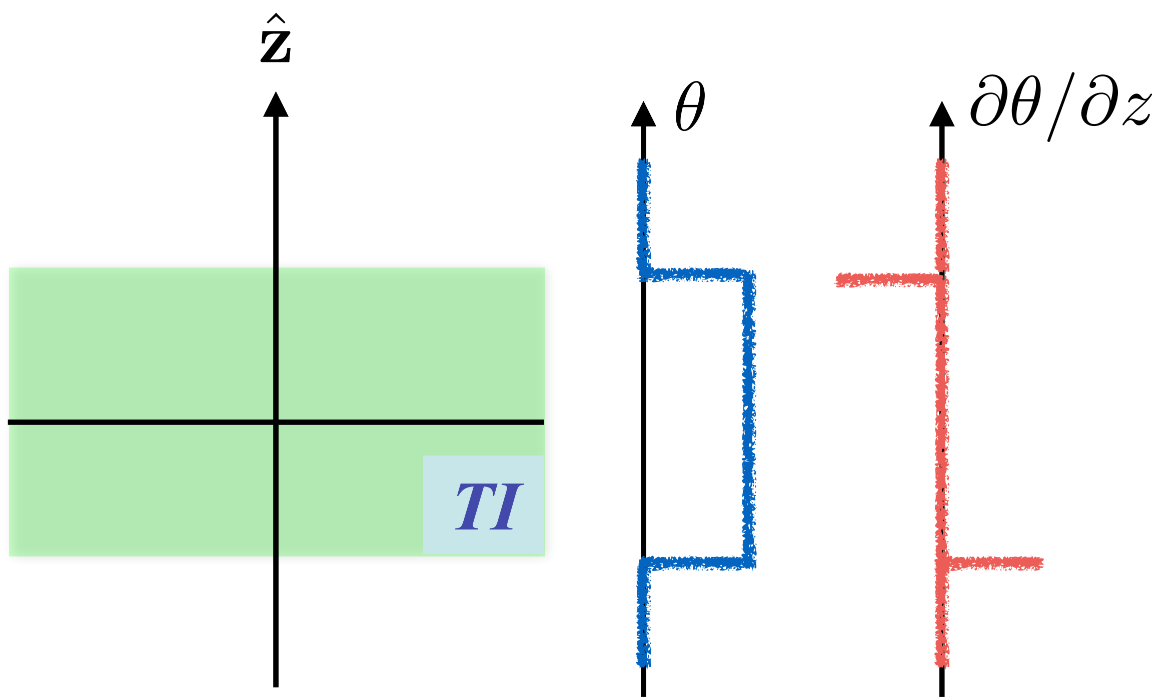
but on the surface of a TI θ changes!



Non-Uniform θ

Gauss' law $$\nabla \cdot \mathbf{E} = 4\pi\rho - \frac{e^2}{\pi}\nabla\theta \cdot \mathbf{B}$$

Ampere's law $$\nabla \times \mathbf{B} = 4\pi\left(\frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}\right) + \frac{e^2}{\pi}\left(\nabla\theta \times \mathbf{E} + \frac{\partial \theta}{\partial t}\mathbf{B}\right)$$



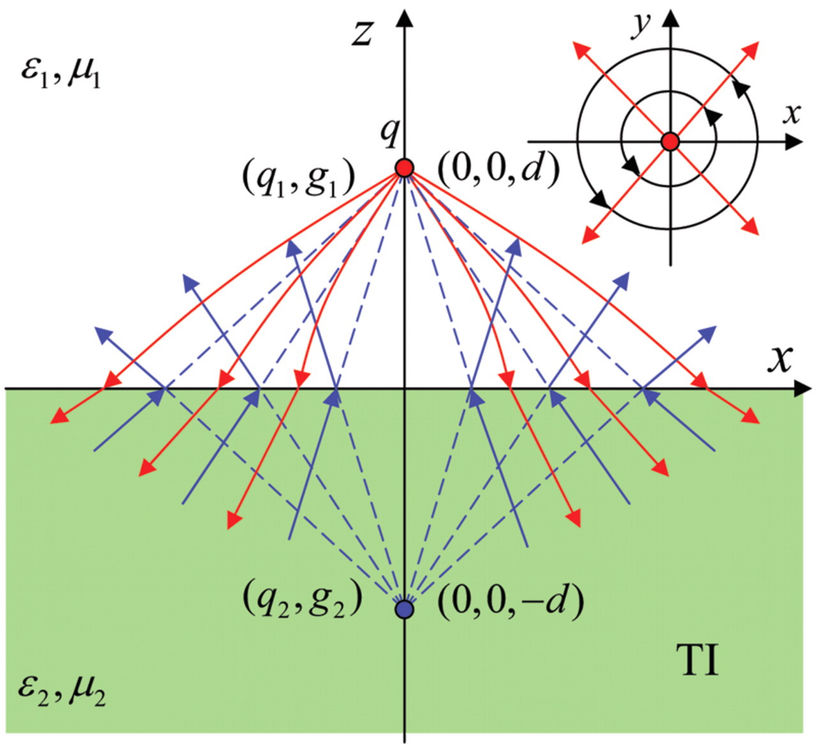
Non-uniform θ : induced magnetic monopole

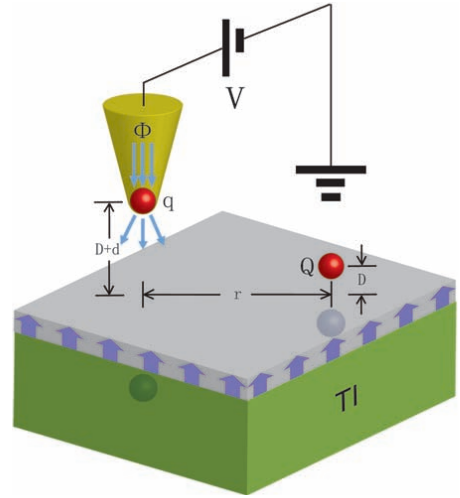
Qi, Li, Zang & Zhang,
Science 323, 1148 (2009)

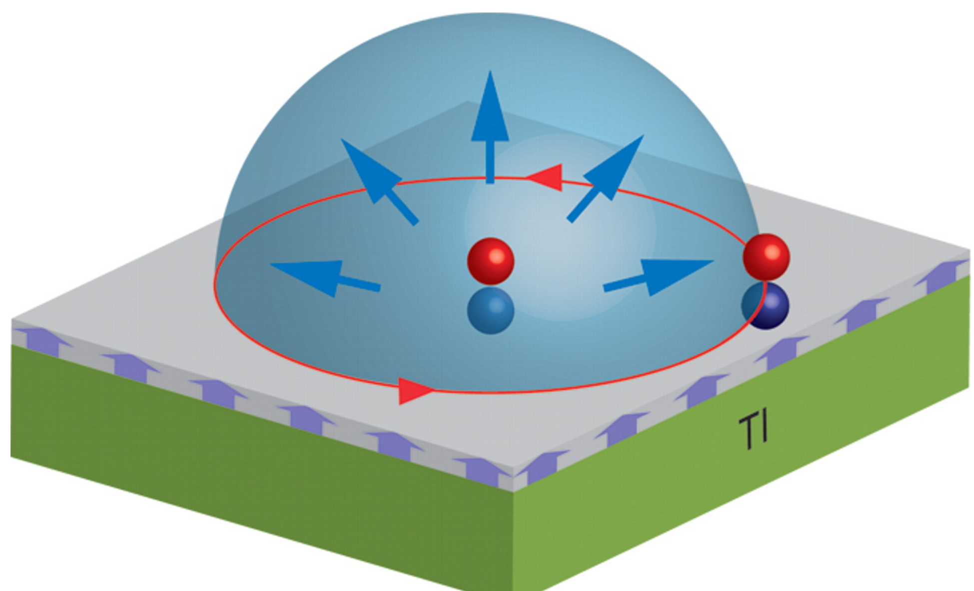
Inducing a Magnetic Monopole with Topological Surface States

Xiao-Liang Qi,[1] Rundong Li,[1] Jiadong Zang,[2] Shou-Cheng Zhang[1]*

Existence of the magnetic monopole is compatible with the fundamental laws of nature; however, this elusive particle has yet to be detected experimentally. We show theoretically that an electric charge near a topological surface state induces an image magnetic monopole charge due to the topological magneto-electric effect. The magnetic field generated by the image magnetic monopole may be experimentally measured, and the inverse square law of the field dependence can be determined quantitatively. We propose that this effect can be used to experimentally realize a gas of quantum particles carrying fractional statistics, consisting of the bound states of the electric charge and the image magnetic monopole charge.







where do these "monopoles" come from?

what about $\nabla \cdot \mathbf{B} = 0$ *everywhere?*

Authors use 2 image charges and 2 image monopoles

For d=0 a direct solution of field equations can be obtained

$$\mathbf{A}(\mathbf{r}, z) = \frac{\alpha\theta}{4\pi^2} \int d^2r' \frac{\hat{\mathbf{z}} \times \mathbf{E}(\mathbf{r}', z'=0)}{\sqrt{(\mathbf{r}-\mathbf{r}')^2 + z^2}}$$

$$\begin{aligned}\mathbf{B}(\mathbf{r}, z) &= \nabla \times \mathbf{A} = \frac{\alpha\theta}{4\pi^2} \left\{ z \int d^2r' \frac{\mathbf{E}(\mathbf{r}', z'=0)}{[(\mathbf{r}-\mathbf{r}')^2 + z^2]^{3/2}} \right. \\ &- \left. \hat{\mathbf{z}} \int d^2r' \frac{(\mathbf{r}-\mathbf{r}') + z\hat{\mathbf{z}}}{[(\mathbf{r}-\mathbf{r}')^2 + z^2]^{3/2}} \cdot \mathbf{E}(\mathbf{r}', z'=0) \right\}\end{aligned}$$

$\Longrightarrow$ Magnetic field satisfies $\nabla \cdot \mathbf{B} = 0$

Nogueira & JvdB, arXiv:1808.08825

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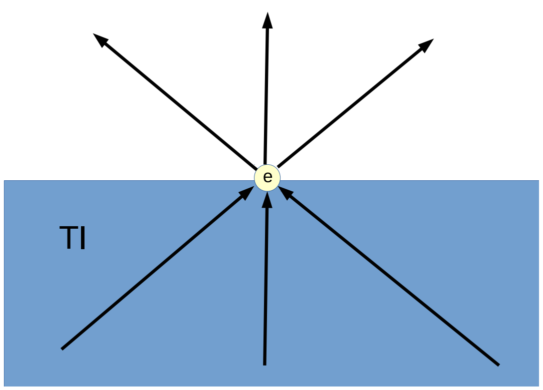
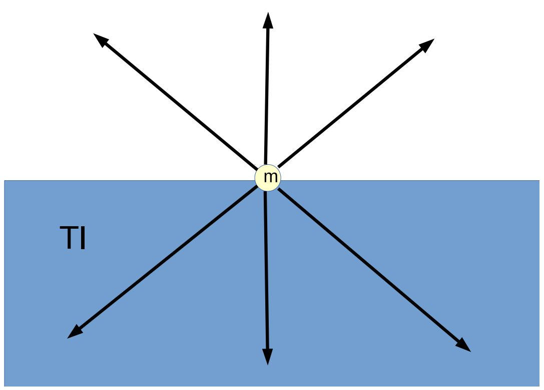


Point vortex



Monopole

Nogueira & JvdB, arXiv:1808.08825

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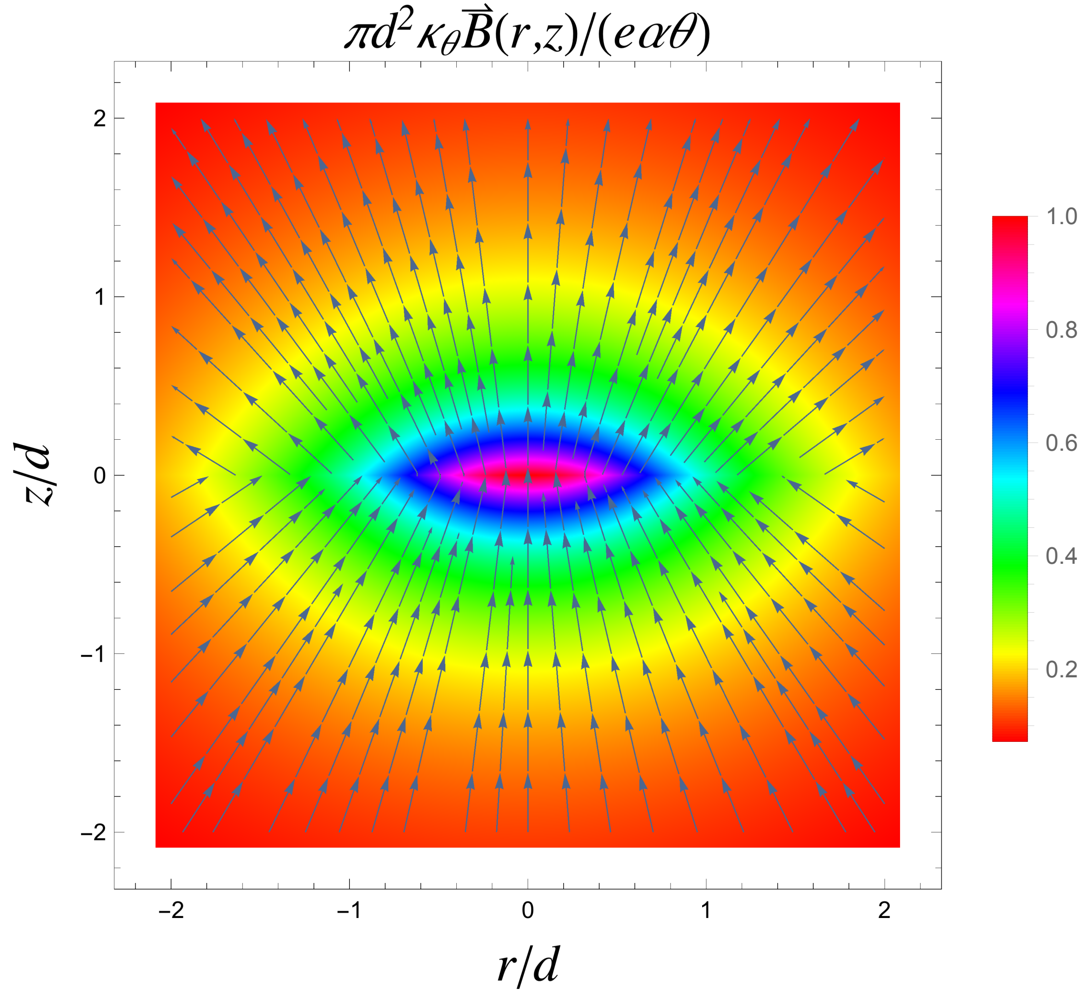
Direct solution for finite d



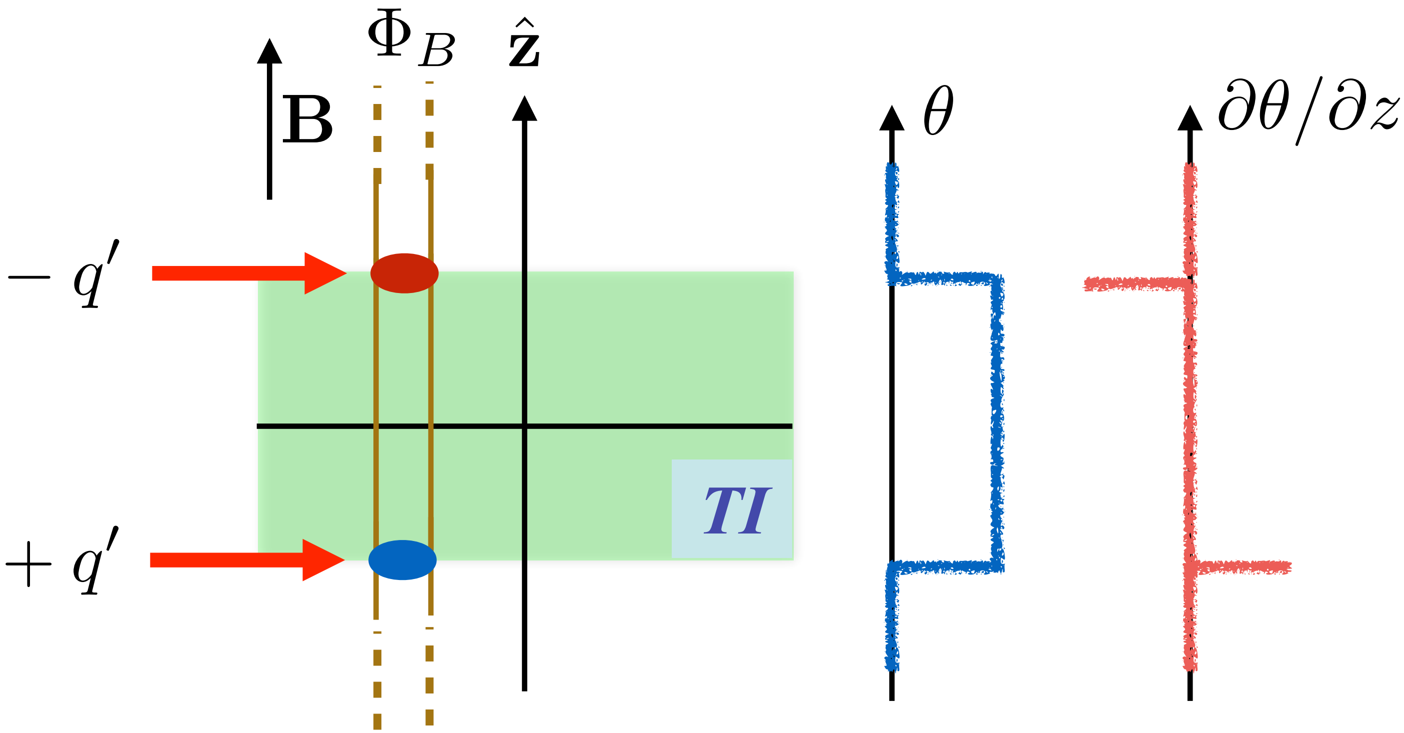
Nogueira & JvdB,
arXiv:1808.08825

Non-uniform θ: magnetic solenoid with flux Φ_B

Gauss' law $$\nabla \cdot \mathbf{E} = 4\pi\rho - \frac{e^2}{\pi}\nabla\theta \cdot \mathbf{B}$$

total charge $$\rightarrow Q = q + \frac{e^2}{4\pi^2}\int d^2r B(r) \int dz \frac{d\theta}{dz} = q + \frac{e^2\theta}{4\pi^2}\Phi_B$$

MAGNETIC FLUX INDUCES CHARGE $$= q + q'$$

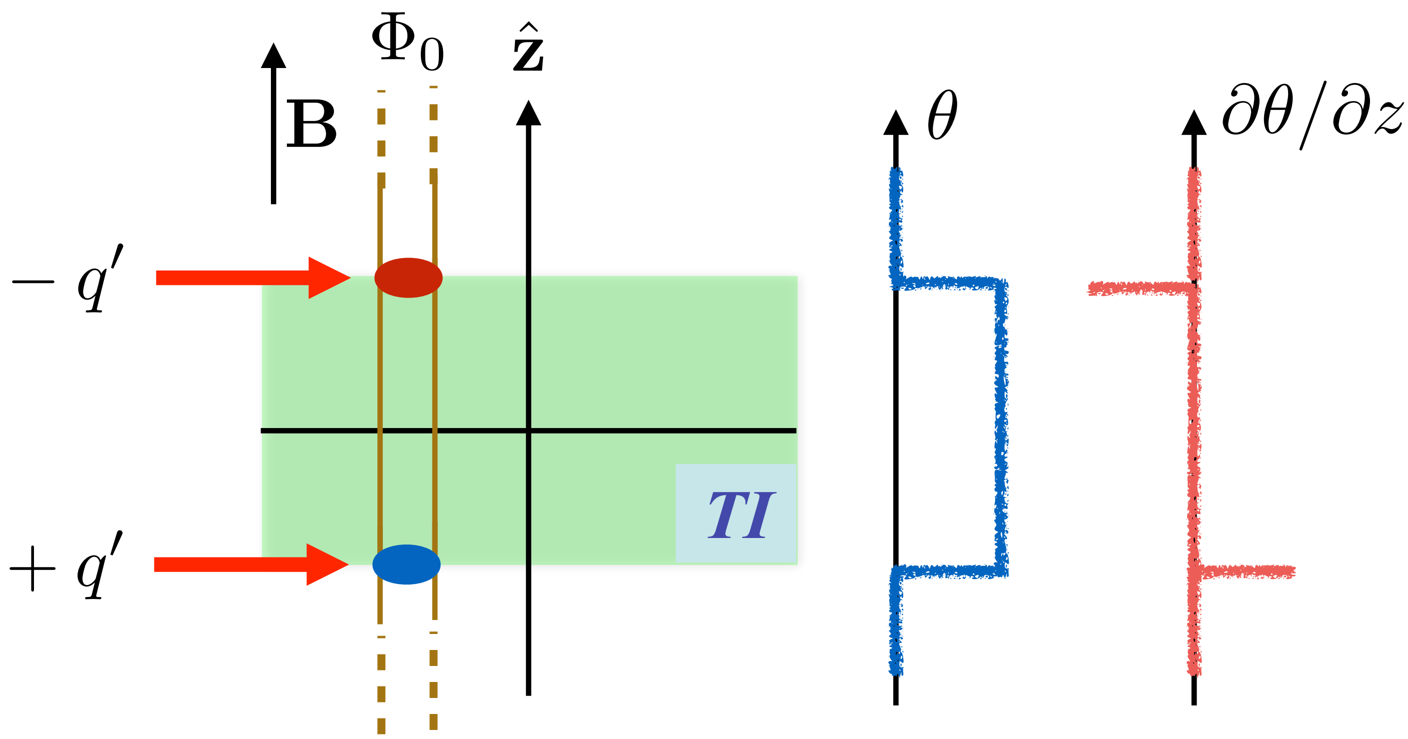


Non-uniform θ: magnetic solenoid with flux Φ_0

Vortex in superconductor carries flux quantum $\Phi_0 = \pi/e$

so that $q' = \frac{e^2\theta}{4\pi^2}\Phi_0 = \frac{e\theta}{4\pi}$ *and for a TRI 3D TI* $q' = \frac{e}{4}$

CHARGE FRACTIONALIZATION



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CHARGE FRACTIONALIZATION

WITTEN EFFECT
WITHOUT MONOPOLES

Vortex line can be viewed as world-line of a magnetic monopole (duality)

How can a vortex line enter a strong TI?

Nogueira, Nussinov & JvdB, PRD 94, 085003 (2016)

Nogueira, Nussinov & JvdB, PRL 117, 167002 (2016)

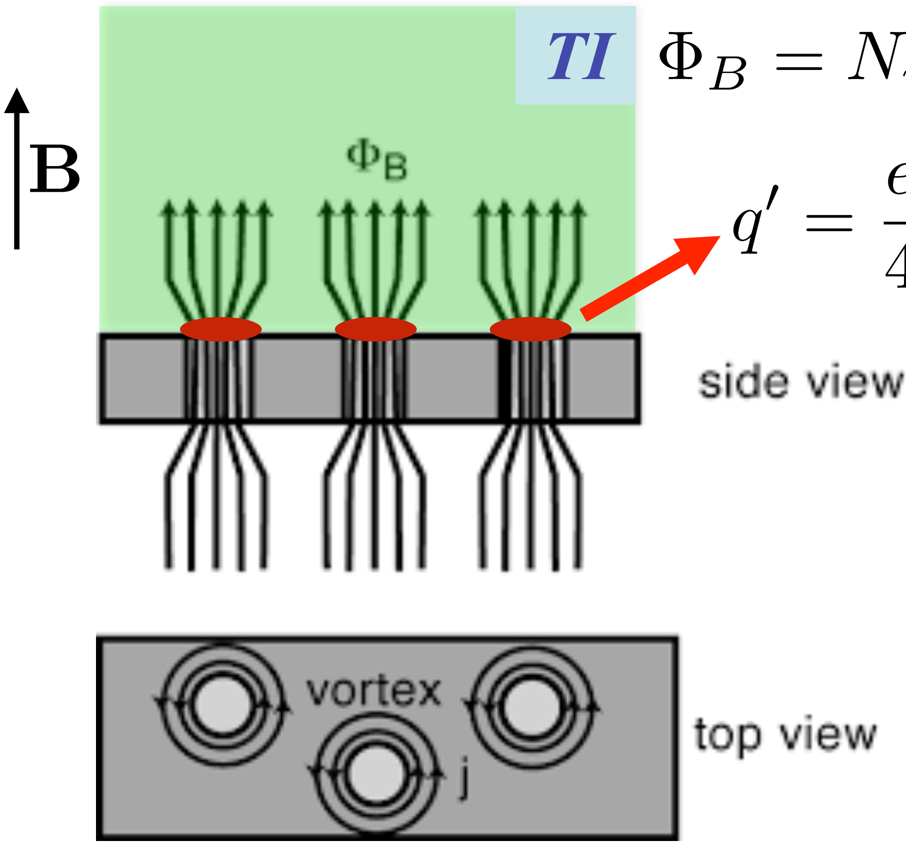
Vortex in strong TI

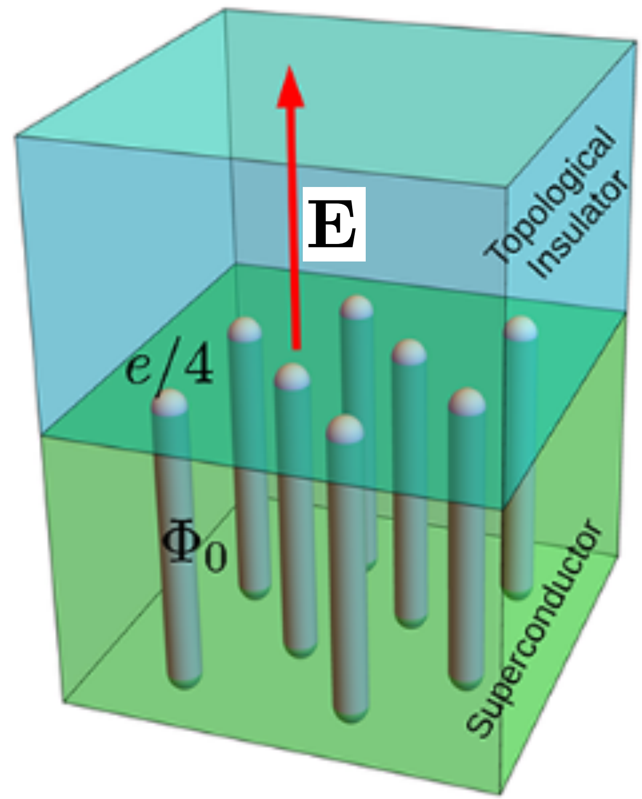
Type II superconductor in $\mathbf{B}$ *field*

in junction with 3D TI

$$\Phi_B = N_v \Phi_0$$

$$q' = \frac{e}{4}$$



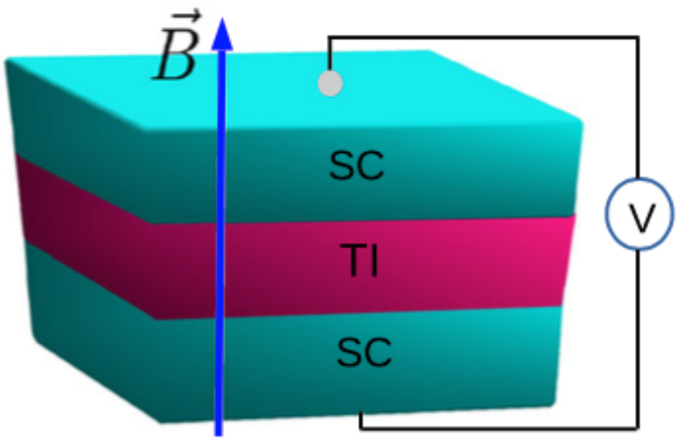


number of vortices N_v

Does NOT require SC proximity effect!

Josephson effect: SC-TI-SC junction

SC-TI-SC junction



$$\Delta V_{\text{ind}} = \frac{2e}{C}\left(\Delta n + \frac{\theta}{8\pi} N_v\right)$$

even without external potential:

B *field induces AC Josephson effect*

due to axion term in EM Lagrangian $\mathcal{L}$

→ *Josephson-Witten effect*

In presence of additional AC voltage $V_t = V_0 + V_1 \cos\omega_1 t$

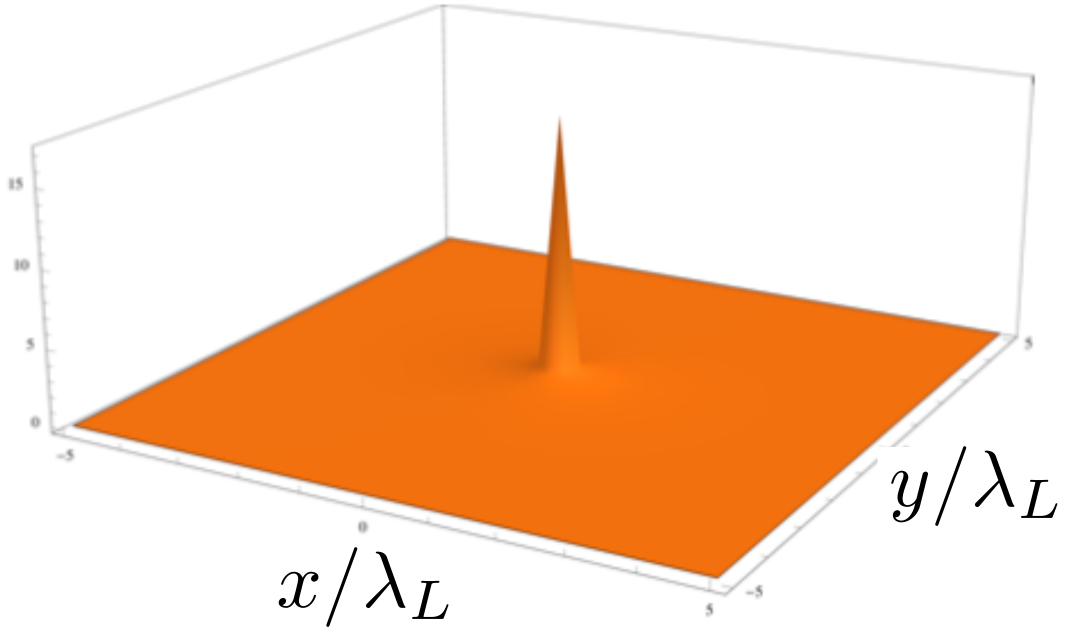
Shapiro steps at $V_0 = V_{nm} = \dfrac{n\omega_1}{2e} - \dfrac{2me}{C}$

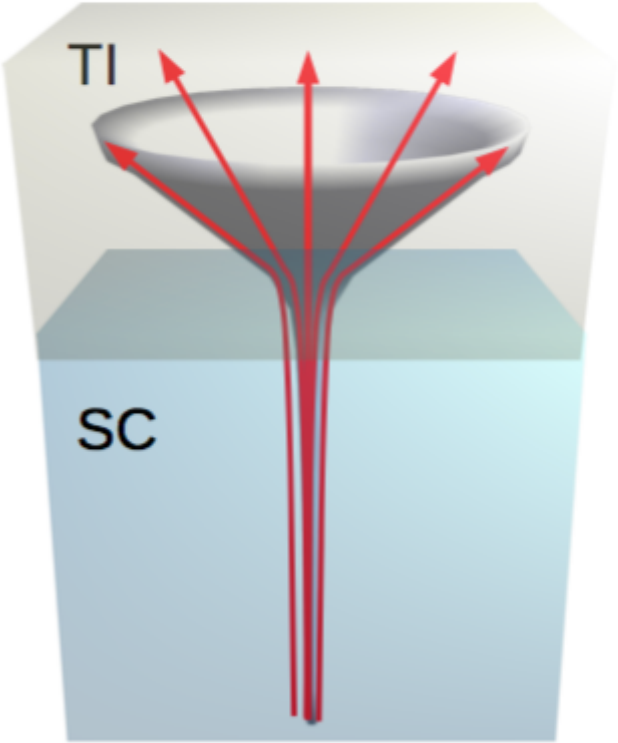
"Charge lattice"

Charge distribution e/4 charge @vortex

Solve Maxwell equations with appropriate boundary conditions

Calculated surface charge density





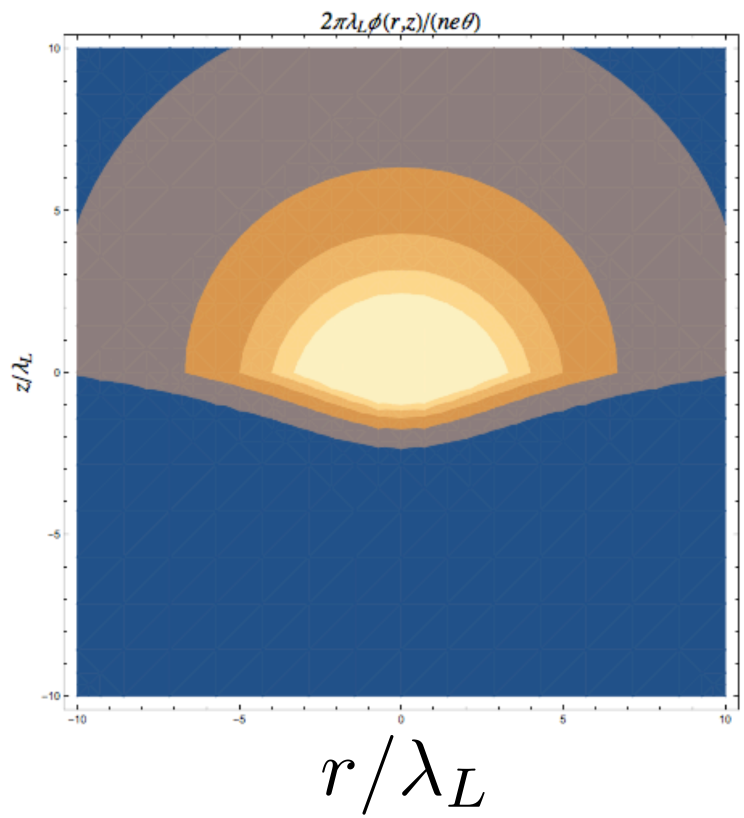
London penetration depth λ_L

Dielectric constant $\epsilon = 100$

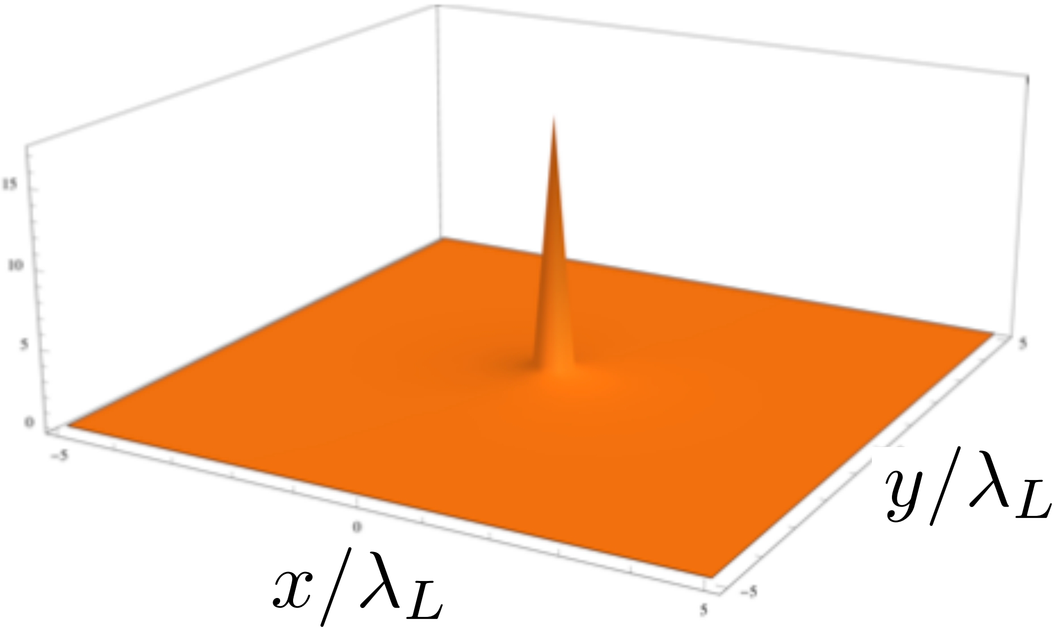
Nogueira, Nussinov & JvdB,
PRL 121, 227001 (2018)

Charge distribution e/4 charge @vortex

electric potential



Calculated surface charge density



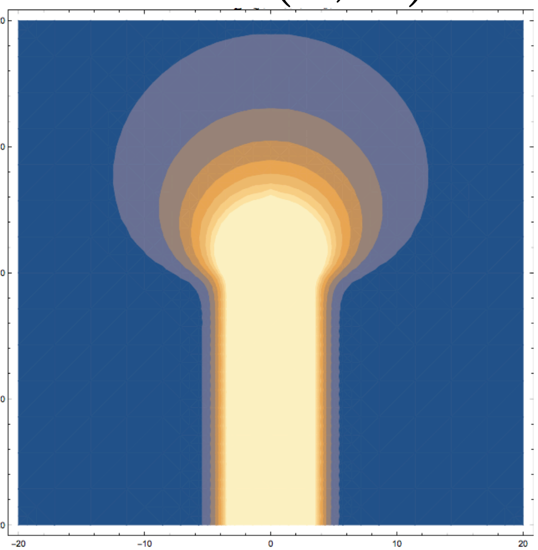
London penetration depth λ_L

Dielectric constant $\epsilon = 100$

B @vortex

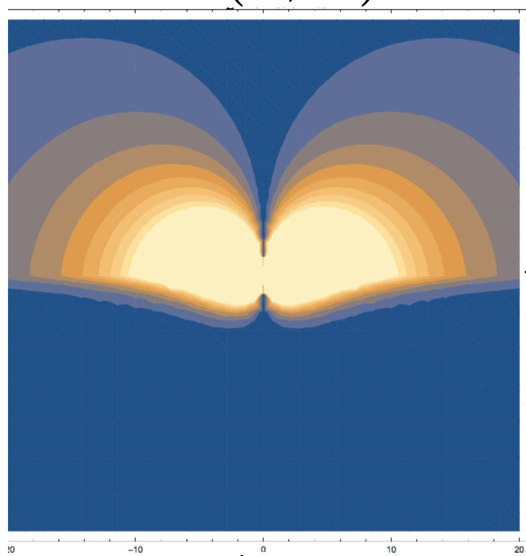
Magnetic field $\mathbf{B}$

$B_z(r,z)$



r/λ_L

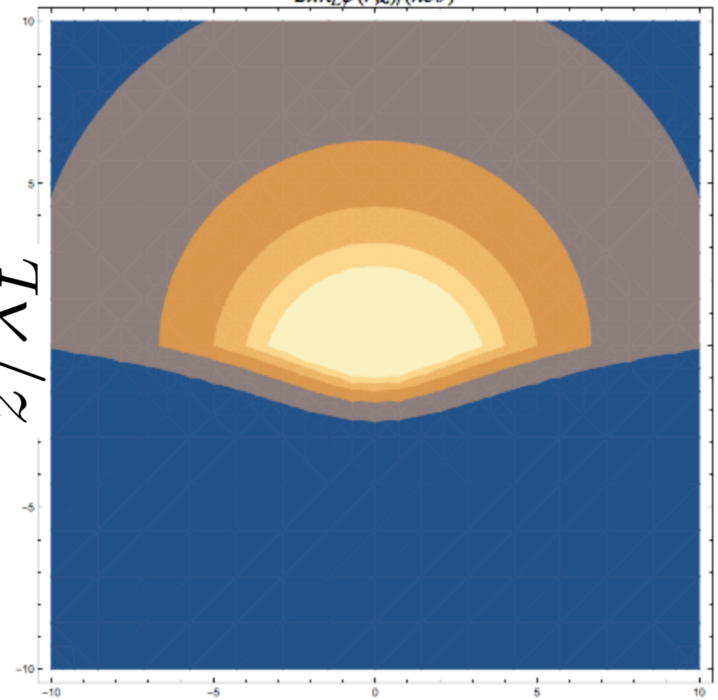
$B_r(r,z)$



r/λ_L

electric potential

$2\pi\lambda_L\phi(r,z)/(ne\theta)$



z/λ_L

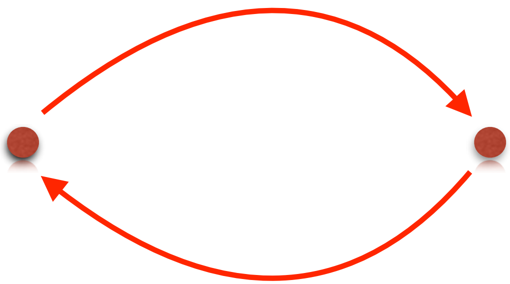
r/λ_L

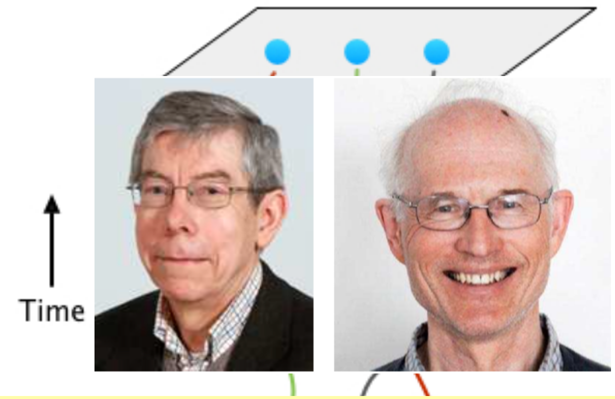
London penetration depth λ_L

Dielectric constant $\epsilon = 100$

Exchange two particles 2D

$$P_{12}\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\gamma}\psi(\mathbf{r}_2,\mathbf{r}_1)$$





Leinaas & Myrheim
Nuovo Cimento B. 37, 1 (1977)

statistical angle γ can take any value

$\rightarrow$ anyon

rather than exchanged (permuted), anyons are braided

extremely robust

topologically protected

How to construct anyons?

introduce charged particles & attach magnetic flux

Φ can take any value $\rightarrow$ anyon

Wilczek PRL 957 (1982)

Unfortunately does not work for Maxwell’s electromagnetic fields

Jackiw & Redlich PRL 555 (1983)

Need emergent fluxes and/or charges

Need topological term in Lagrangian

Angular momentum of fractionally charged vortex

Total angular momentum

$$\mathcal{J} = \mathbf{J} + \mathbf{L}$$

$\mathbf{L}$ → mechanical

$\mathbf{J}$ → field

$$\mathbf{J} = \frac{1}{4\pi c}\int d^3R(\nabla\cdot\mathbf{E})(\mathbf{R}\times\mathbf{A})$$

because of symmetry $\mathcal{J} = \mathcal{J}_z\hat{\mathbf{z}}$

Nogueira, Nussinov & JvdB,
PRL 121, 227001 (2018)

Angular momentum of fractionally charged vortex

Total angular momentum

$$\mathcal{J}_z = -\frac{n^2 \hbar \theta}{8\pi} = \frac{n \Phi_0 Q}{2\pi c}$$

Statistical angle

$$\gamma = 2\pi J_z / \hbar \qquad = \pi/4$$

for an elementary flux quantum at the surface of a time reversal invariant TI

$\gamma = 0$ boson

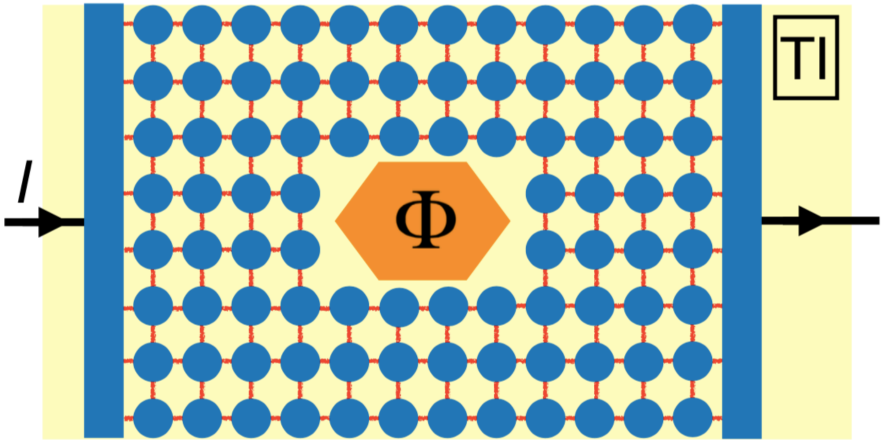
$\gamma = \pi/4$ quadrantion ???

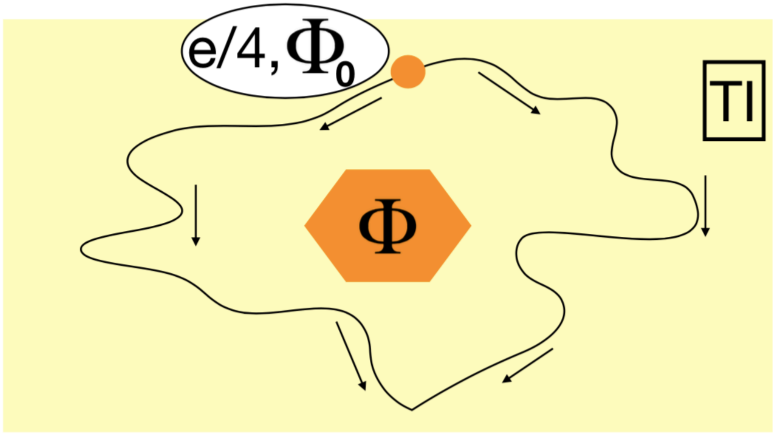
$\gamma = \pi/2$ semion

$\gamma = \pi$ fermion

Measuring fractionally charged vortex

Josephson junction array on TI





tune to flux-flow regime

vortex interference

Aharanov-Bohm phase

8 π periodicity of differential conductance

Nogueira, Nussinov & JvdB, PRL 121, 227001 (2018)

Conclusions

$\nabla \cdot \mathbf{B} = 0$ *not violated in axion electromagnetic response*

Witten effect provides topological mechanism to induce AC Josephson effect when B-field perp to SC-TI-SC junction

Induced Josephson frequency is quantized

None of this requires SC proximity effect

Vortices at TI surface are anyons

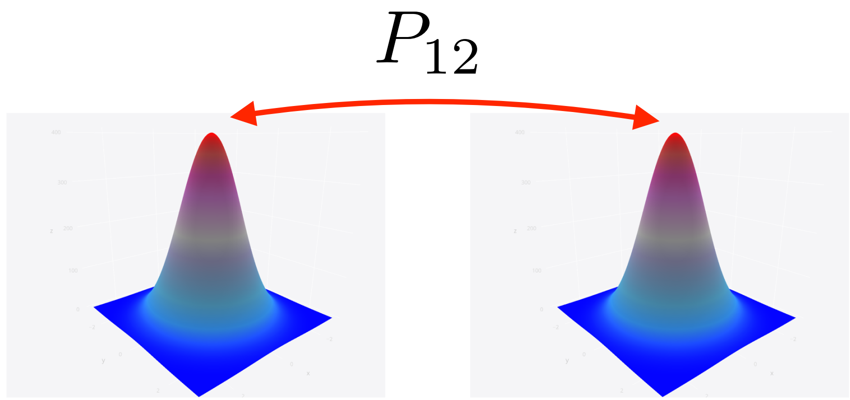
with statistical angle $\gamma = \pi/4$

Flavio Nogueira

Zohar Nussinov

What is quantum statistics of 2 charged vortices?

exchange operator of the two particles in 3D



wavefunction $\psi(\mathbf{r}_1, \mathbf{r}_2)$

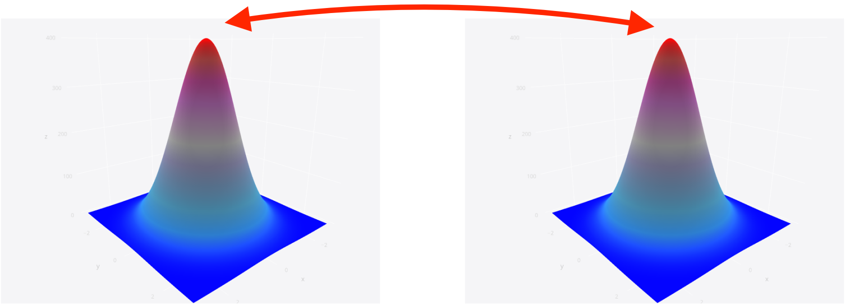
as $P_{12}^2 \psi(\mathbf{r}_1, \mathbf{r}_2) = \psi(\mathbf{r}_1, \mathbf{r}_2)$

it follows that
$$P_{12}\psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\psi(\mathbf{r}_2, \mathbf{r}_1)$$
$$= e^{i\gamma}\psi(\mathbf{r}_2, \mathbf{r}_1) \qquad \gamma = 0, \pi$$

Quantum statistics of 2 particles in 3D

exchange operator of the two a particles

$$P_{12}$$



bosons $P_{12} = +1$ integer intrinsic angular momentum

fermions $P_{12} = -1$ half integer intrinsic angular momentum